Advanced Macroeconomics I

Lecture 8 Business Cycles

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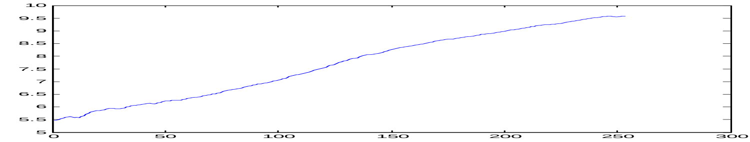
The Behavior of Economic Aggregates

- Growth theory: output grows secularly
- Business cycle theory: it fluctuates around its long term trend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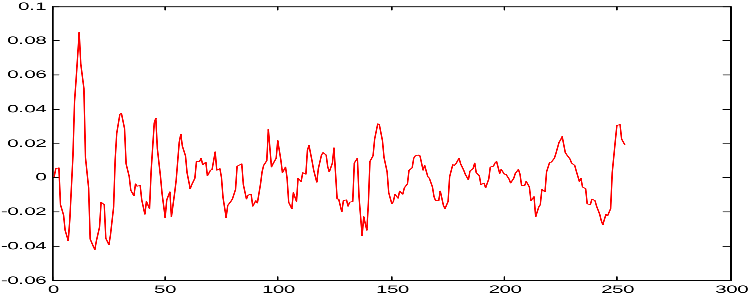
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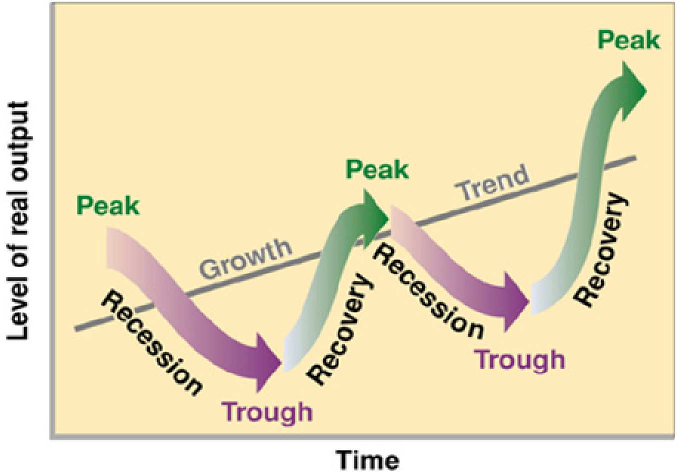
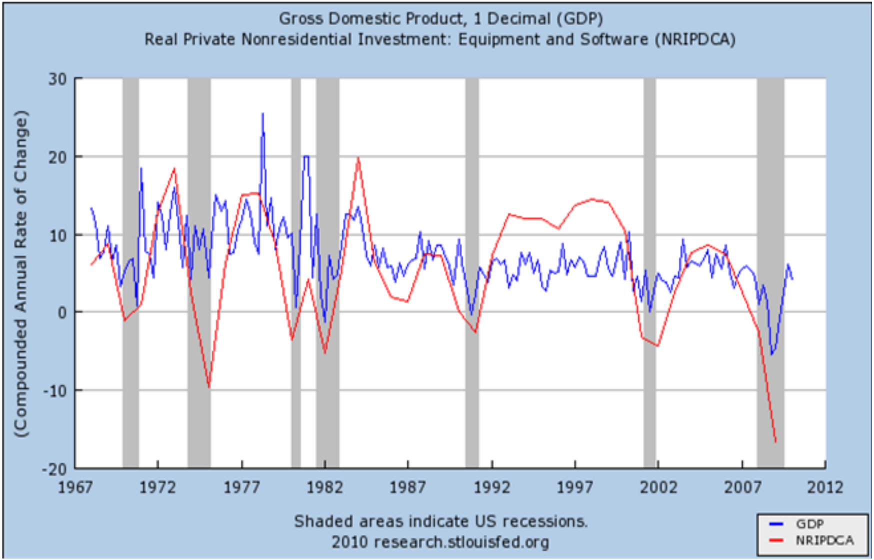
Business Cycles



Business Cycles



Business Cycles

- Facts and questions
  - Why are there cycles (what shocks)
  - How do they work (how do they propagate)
- Attempts at explaining these facts
  - Keynesianism
  - Real Business Cycles
  - New Keynesian models
  - Sunspot theories

Keynesianism

- Postulating that investors were driven by "animal spirits"
  - impulse feelings on consumption decision
  - lack of rationality; hence the government is called upon to correct this behavior
  - Lucas' critique (1976, 1977): real business cycles

- Kydland and Prescott (1982), and Long and Plosser (1983)
  - There are technology shocks that affact the productivity of factors
  - The source of the shock is real, and the propagation mechanism is real as well: it is a consequence of the intertemporal substitution of labor that optimizing decision makers choose whenever confronted with such a technology shock
- The critique to this approach is that the definition of a technology shock is somewhat blurred. What is exactly such a shock?

- The reason why this tradition has focused on the "real" explanation of business cycles is rather accidental
  - When Prescott undertook the research program laid down by Lucas (1977) paper, the initial schedule was to start with a real source of the cycle (the technology shock) and the real propagation mechanism (the inter-temporal substitution), thereafter to increase the complexity of the model and allow for monetary mechanisms
- However, on seeing that the real approach was providing a seemingly thorough explanation of the phenomenon, the course of the research program deviated towards increasing the complexity and richness of the real setup (such as introducing heterogeneity in agents, incomplete markets, etc.)

New Keynesian

- Opposed to the real approach, these take a monetary approach
  - The source of the cycles are monetary fluctuations
  - The main propagation mechanism is also monetary: price "stickiness"

Sunspot theories

- Micro foundations models in which agents have full rationality
- But are faced with models that have multiple equilibria
  - This allows for self-fulfilling, rational expectations that may cause fluctuations of output, even in spite of the absence of an underlying change in the production or utility fundamentals in the economy
- This can be interpreted as a coordination problem, in which agents fail to achieve the "best" equilibrium out of the available ones
- Notice that to some extent, the "animal spirits" (or consumer confidence) concept can be accommodated to explain why agents simultaneously believe that a given equilibrium will be realized, and act accordingly

Facts

- What the economics profession regards as the acceptable indicators to be drawn from that data
- What is the correct methodology to transform raw data into acceptable "facts"?

Early Facts by Burns and Mitchell (1946)

- Output in different sectors of the economy have positive covariance
- Both investment and consumption of durable goods exhibit high variability
- Company profits are very pro-cyclical and variable
- Prices are pro-cyclical as well (This is not true for the post-war years, but it was for the sample analyzed by Burns and Mitchell)
- The amount of outstanding money balances and the velocity of circulation are procyclical
- Business cycles are "all alike" across countries and across time

Critique

- Burns and Mitchell's work was harshly criticized by Koopmans (1947)
  - The work was not carefully done, and was hard to replicate
  - There was no solid underlying statistical theory. Relevant issues were not addressed altogether, such as the statistical significance of the assertions
- Koopmans' counter-argument discredited Burns and Mitchell's approach to the extent that no literature developed to improve and further their view

"macroeconometrics" approach

- Klein (Nobel prize due to this research) and Goldberg
- Yale's Cowles commission: studying huge econometric models of macroeconomic variations
- However, little has been left behind by this methodology, which ended up consisting of building up large scale macroeconomic models, making them bigger and bigger variable-wise until the regressions explained something
- Lucas' critique (Lucas (1976)) that found widespread agreement through the economic profession, put an end to this research program

Kydland and Prescott (1982):

How to convert raw data into facts

- Kydland and Prescott's work is the founding stone of the current consensus on what "facts" are
- These authors went back to the Burns and Mitchell tradition of stylized facts, but provide a solid methodological foundation for their approach
- Raw data need to be rid of the secular growth component before the cycle can be identified. This is done by filtering the data, using the method developed by Hodrick and Prescott (the so-called "HP filter")

HP filter

- The HP filter procedure to de-trend data is to solve the following minimization problem:

$$\min_{\{\bar{y}_t\}_{t=1}^{T}} \left\{ \sum_{t=1}^{T} (y_t - \bar{y}_t)^2 \right\}$$

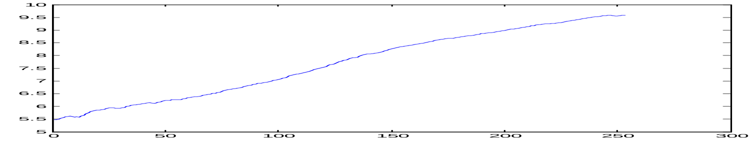
$$\sum_{t=2}^{T-1} \left[(\bar{y}_{t+1} - \bar{y}_t) - (\bar{y}_t - \bar{y}_{t-1}) \right]^2 \leq K$$

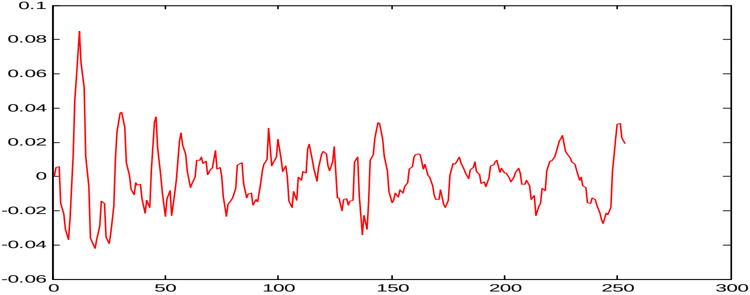
- In practice, K is set equal to 0, and this leads to the following Lagrangian:

$$\begin{aligned} L &= \sum_{t=2}^{T-1} \left\{ (y_t - \bar{y}_t)^2 - \mu \left[(\bar{y}_{t+1} - \bar{y}_t) - (\bar{y}_t - \bar{y}_{t-1}) \right]^2 \right\} \\ &\quad + (y_T - \bar{y}_T)^2 + (y_1 - \bar{y}_1)^2 \end{aligned}$$

- Hodrick and Prescott chose $\mu = 1600$ to de-trend quarterly data, and $\mu = 400$ for annual data

Business Cycles-HP filter





Kydland and Prescott's facts

- Object of study is the resulting $(y_t - \bar{y}_t)^2$ sequence. With this in hand, "facts" in business cycles research are a series of relevant statistics computed from de-trended data
- 1. Volatilities
- 2. Correlations
- 3. Persistence

Volatilities

- Given a variable x; we define its percentage standard deviation as:

$$\sigma_x \equiv \frac{[Var(x)]^{1/2}}{\mu_x}$$

μ_x denotes the mean value of x

- $\sigma_c < \sigma_y$: interpreted as consumption smoothing
- $\sigma_{c_o} < \sigma_y$: c_o consumption durables
- $\sigma_i \approx 3\sigma_y$; $\sigma_K < \sigma_y$
- $\sigma_N \approx \sigma_y$; $\sigma_E \approx \sigma_y$
- $\sigma_w \approx \sigma_{N/y}$: real wage is sticky

Correlations and Persistence

Correlations

- $\rho(y/N,\ y) > 0$
- $\rho(w, y) \approx 0$: Recall that y/N is the average product of labor, and w is the marginal product
- $\rho(K, y) \approx 0$
- $\rho(P, y) < 0$ (in post-war period)

Persistence

- $\rho((y_t - \bar{y}_t), (y_{t-1} - \bar{y}_{t-1})) \approx 0.9$ (quarterly data)

Stochastic Neoclassical Growth Model

- The success of a real business cycle model is measured by its ability to numerically replicate the "facts"
- The basic model is the planner's problem to optimize the use of resources according to a time-additive preference relation that admits a utility function representation. For example, if production is affacted by a shock on total factor productivity that follows an AR(1) process, the problem is:

$$\max_{\{c_t,\ n_t,\ l_t,\ K_{t+1}\}_{t=0}^{\infty}} \left\{ E_0 \left[\sum_{t=0}^{T} \beta^t u\left(c_t,\ l_t\right) \right] \right\}$$

$$\begin{aligned} s.t.\quad c_t + x_t &= z_t F\left(K_t, n_t\right) \\ K_{t+1} &= \left(1-\delta\right) K_t + X_t \\ l_t + n_t &= 1 \\ z_{t+1} &= \rho z_t + \varepsilon_{t+1} \end{aligned}$$

Calibration

- Calibration requires that values for parameters be picked from sources independent of the phenomenon under study
- Calibration bans "curve fitting": contrast to Macroeconomitrics
- Example of admissible data:
  - - Household data on consumption, hours worked, and other microeconomic evidence, for individual preference parameters
  - Long run trend data for the factor shares in production (namely α in the Cobb-Douglass case)

Steps to follow in real business cycle research

- Specify a model, including functional forms and parameters
- Pick parameters through calibration
- Solve the model numerically
- Simulate the model and analyze the outcome

Solve the model

- Most often, this will be done using linearization methods. Recall that in order to do this, given an AR(1) process for the stochastic shock:

$$z' = \rho z + \varepsilon$$

- the policy rule guesses were linear in the state variables (K, z):

$$K' = a_K + b_K K + d_K z$$

$$n = a_n + b_n K + d_n z$$

- The task is to solve for the parameters $a_K, a_n, b_K, b_n, d_K, d_n$

Simulate the model and analyze the outcome

- A random number generator is used to simulate a realization of the stochastic shock
- This gives rise to a time series in each of the variables
- These series are the researcher's "data set"
- Sample moments of the variables (in general, second moments) are computed and compared to actual data

An example of RBC

- Preference

$$u(c, l) = \frac{\left(c^{1-\theta} l^{\theta}\right)^{1-\sigma} - 1}{1-\sigma}$$

- The size of household's population grows at rate η
- The centralized formulation of the utility maximization problem is:

$$\max_{\{c_t,\ l_t\}_{t=0}^{\infty}} \left\{\beta^t (1+\eta)^t u(c_t, l_t)\right\}$$

$$s.t. \quad C_t + X_t = A(1+\gamma)^{t(1-\alpha)} K_t^{\alpha} N_t^{1-\alpha}$$

- γ labor-augmenting technology

Per capita term equations

- Per capita term resource constraint

$$\frac{C_t}{P_t} + \frac{X_t}{P_t} = A(1+\gamma)^{t(1-\alpha)} \left(\frac{K_t}{P_t}\right)^{\alpha} \left(\frac{N_t}{P_t}\right)^{1-\alpha}$$

Population P_t

$$c_t + x_t = A(1+\gamma)^{t(1-\alpha)} k_t^{\alpha} n_t^{1-\alpha}$$

- Law of Motion of capital per capita

$$K_{t+1} = (1-\delta) K_t + X_t$$

$$(1+\eta) \frac{K_{t+1}}{P_t (1+\eta)} = (1-\delta) \frac{K_t}{P_t} + \frac{X_t}{P_t}$$

$$(1+\eta) k_{t+1} = (1-\delta) k_t + x_t$$

Transform the growth model into a stationary one

- De-trended variables
  growth rate γ

$$\tilde{c}_t = \frac{c_t}{(1+\gamma)^t}, \tilde{x}_t = \frac{x_t}{(1+\gamma)^t}, \tilde{k}_t = \frac{k_t}{(1+\gamma)^t}$$

- We specify $\sigma = 1$

$$\log c = \log \tilde{c} + \log (1+\gamma)^{-t}$$

- $$\max_{\{c_t,\ l_t\}_{t=0}^{\infty}} \left\{ \beta^t (1+\eta)^t \left[\log \tilde{c}_t + \frac{\theta}{1-\theta} \log l_t \right] \right\}$$

$$\tilde{c}_t + (1+\eta)(1+\gamma)\tilde{k}_{t+1} == A\tilde{k}_t^{\alpha} (1-l_t)^{1-\alpha} + (1-\delta)\tilde{k}_t$$

Parameters to be Calibrated

	interpretation	target
α	capital share	$\frac{rK^*}{Y^*}$
δ	depreciation rate	$\frac{X^*}{K^*}$(law of motion)
γ	growth rate	growth rate
A	average productivity	irrelavent
σ	elasticity of intertemporal substition	1
θ	preference share of consumption	$\frac{Y^*}{K^*}$(f.o.c. k_{t+1})
β	discounting factor	$\frac{Y^*}{C^*}$(f.o.c. c, l)
η	population growth rate	P growth rate

Parameters

- Immediate available parameters:
  - $\alpha = 0.4$ (capital share of output - constant)
  - $\gamma = 0.2$ (average long run growth rate)
  - $\eta = 0.01$ (average long run population growth rate)
  - A is a scaling factor. It is irrelevant

Parameters

- δ can be found in the following way. In the steady state of the transformed model (i.e. on the balanced growth path), we have that

$$\tilde{k}_{t+1} = \tilde{k}_t = \tilde{k}^*$$

- Capital accumulation equation

$$(1+\eta)(1+\gamma)\tilde{k}^* = (1-\delta)\tilde{k}^* + \tilde{x}^*$$

$$\begin{aligned} (1+\eta)(1+\gamma) &= (1-\delta) + \frac{\tilde{x}^*}{\tilde{k}^*} \\ &= (1-\delta) + \frac{X^*}{K^*} \end{aligned}$$

$$\frac{X^*}{K^*} = 0.076, \ \delta = 0.0458$$

Parameters

- parameter in the utility function: β
- Take first order conditions of the problem. Assuming a balanced growth path ($\tilde{c}_t = \tilde{c}_{t+1}$) we differentiate with respect to next period's capital $\tilde{k}_{t+1}$:

$$1 + \gamma = \beta \left[\alpha A \tilde{k}_t^{\alpha-1} \left(1 - l_t\right)^{1-\alpha} + 1 - \delta \right]$$

$$\alpha A \tilde{k}_t^{\alpha-1} \left(1 - l_t\right)^{1-\alpha} = \alpha \frac{\tilde{y}_t}{\tilde{k}_t} = \alpha \frac{Y^*}{K^*} \approx 0.3012$$

$$\beta = 0.9491$$

Parameters

- Parameter in the utility function: θ
- We need the remaining first order conditions

$$\begin{aligned} \tilde{c}_t &: \quad \frac{1}{\tilde{c}_t} = \lambda_t \\ l_t &: \quad \frac{\theta}{1-\theta}\frac{1-l_t}{l_t} = (1-\alpha)\frac{\tilde{y}_t}{\tilde{c}_t} = (1-\alpha)\frac{Y_t}{C_t} \end{aligned}$$

$$\begin{aligned} \frac{Y_t}{C_t} &= \frac{Y_t}{K_t}\frac{K_t}{C_t} \\ X_t + C_t &= Y_t \\ \frac{X_t}{K_t} + \frac{C_t}{K_t} &= \frac{Y_t}{K_t} \end{aligned}$$

- l_t : use knowledge from microeconomic studies. 8 hours work out of 24 hours, $l_t \approx 2/3$
- $\theta = 0.6161$

Solve the model

- Most often, this will be done using linearization methods. Recall that in order to do this, given an AR(1) process for the stochastic shock:

$$z' = \rho z + \varepsilon$$

- the policy rule guesses were linear in the state variables (K, z):

$$v(K_t, z_t) = \max_{\{c_t, n_t, l_t, k_{t+1}\}_{t=0}^{\infty}} u(c_t, l_t) + \beta E v(K_{t+1}, z_{t+1})$$

$$K' = a_K + b_K K + d_K z$$

$$n = a_n + b_n K + d_n z$$

- The task is to solve for the parameters $a_K, a_n, b_K, b_n, d_K, d_n$

Solve the Model

Optimality conditions:

$$(1+\eta)(1+\gamma)\tilde{k}_{t+1} = (1-\delta)\,\tilde{k}_t + \tilde{x}_t$$

$$(1+\gamma)\frac{1}{\tilde{c}_t} = \beta\frac{1}{\tilde{c}_{t+1}}\left[\alpha z A \tilde{k}_t^{\alpha-1}(1-l_t)^{1-\alpha} + 1 - \delta\right]$$

$$\frac{\theta}{1-\theta}\frac{1-l_t}{l_t} = (1-\alpha)\frac{\tilde{y}_t}{\tilde{c}_t}$$

$$\tilde{c}_t + \tilde{x}_t = \tilde{y}_t$$

$$\tilde{y}_t = zA\tilde{k}_t^{\alpha}(1-l_t)^{1-\alpha}$$

Simulate the model and analyze the outcome

- A random number generator is used to simulate a realization of the stochastic shock
- This gives rise to a time series in each of the variables
- These series are the researcher's "data set"
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