Lumpy Capital, Labor Market Search and Employment Dynamics over Business Cycles

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Abstract

This paper incorporates labor search frictions into a model with lumpy investment to explain a set of firm-size-related facts about the United States labor market dynamics over business cycles. Contrary to the predictions of standard models, we observe that job destruction is procyclical in small firms but countercyclical in large ones. Calibrated to U.S. data, the model generates this asymmetric pattern of employment dynamics in small versus large firms.

In the model, ex ante identical firms face random investment opportunities. Small firms are those that have a fewer number of workers either because they forgo investment for many periods and thus have a lower marginal labor productivity or because they fail to hire workers from a frictional labor market. If a small firm has a low level of capital stock, it tends to make lumpy investment and grow fast. Labor market frictions influence the small firm's investment decisions. If hiring is costly and time consuming, the small firms may give up some investment projects which they would undertake if the labor market were Walrasian. A fovarable aggregate productivity shock tightens the labor market and makes hiring more difficult. A tighter labor market hurts investing small firms, so investment increases more in large firms. As a result, a favorable productivity shock reallocates some workers from small to large firms, making procyclical job destruction in small firms possible. The paper contributes to a better understanding of firms' interactive investment behavior and employment behavior in the presence of factor market frictions, with the resulting aggregate employment dynamics in small versus large firms.

Keywords: labor market search, lumpy capital, business cycle, job creation, job destruction

JEL Classification: E22 E24 E32 E37

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1 Introduction

"In times of recession, large employers disproportionately lose workers, while small companies, as a group, fare better" (Kiviat 2009). Using recent U.S. data this paper documents the different patterns of job flow dynamics in small versus large firms. Small firms are firms with fewer than 250 workers. The empirical regularities are as follows: (1) job creation is procyclical in both small and large firms; (2) job destruction is countercyclical in large firms, but, paradoxically, it is procyclical in small firms; and (3) job creation and job destruction are more volatile in large firms than in small firms.

What might be behind the contrast between small and large firms? Could it be that small firms are less likely to be in the industries hit harder by a recession (such as manufacturing)? Or that small firms are less monitored by profit-seeking shareholders? Given all the instances of small firms growing extremely quickly or disappearing overnight, could it be that workers take the risk to trade for great opportunities in some small firms? Or that small firms are able to hire new talent during a recession because of all the workers being laid off by large firms? All these four factors might be at work to some extent. However, counter to the first factor, the facts still stand out when we look at data for all the private sector firms; and against the second, it is difficult to imagine that small firms are not profit-maximizing to a large extent. So the first two points are unlikely to be the main reason. This paper takes the last two possible reasons mensioned and tells a story in which some small firms grab investment opportunities to grow fast and hunt workers released from large firms during recessions. This story is delivered by a model that combines lumpy investment and labor search frictions.

A full understanding of employment dynamics requires that we account for these different patterns of job flow dynamics. And, given the large share of national income represented by labor, the above facts are also important for a proper understanding of business cycles. Yet, standard models find it difficult to explain these facts. In a standard real business cycle model, for example, the size of a firm is not determinate. In an augmented real business cycle model where firms differ in productivity and labor market is frictional, as in Veracierto (2007), small firms and large firms behave similarly. The different patterns in small versus large firms suggest that there is a force that causes asymmetric behavior in small versus large firms, and this force changes over business cycles.

We look at the real frictions a firm may face and explore how these frictions may affect small firms and large firms differently. Both capital and labor market frictions are documented to be important for firms' interactive investment and employment behavior (Contreras 2007). We find that the interaction between labor search and lumpy investment decisions provides a mechanism to account for the facts listed above.

The simple mechanism works as follows. A new investment project requires new workers to operate it. The investment decision is therefore affected by labor market search frictions. Firms' investment opportunities come at random. Those firms that forgo investment for many periods are smaller and more likely to make new and lumpy investment. If they macke an investment, small firms will also need more workers and will therefore be more affected by labor market search frictions. A favorable aggregate productivity shock tightens the labor market. A tighter labor market hurts investing small firms the most, encouraging the investment by large firms. As a result, the aggregate productivity shock changes the composition of workers in small and large firms.

A similar composition effect is shown in Acemoglu (2001), in which a tighter labor market hurts good jobs (that require more investment in capital and therefore have higher labor productivity) and encourages the creation of bad jobs. While in Acemoglu (2001) the tighter labor market is caused by too much creation of bad jobs in an inefficient equilibrium, in this paper it is caused by a favorable productivity shock. In other words, business cycle shocks change labor market tightness and, therefore, the relative returns of a large amount of investment and a small amount of investment. The fast growing small firms are the most affected.

During recessions, the opposite is the case. After losing workers to large firms during booms, small firms experience self-correction during recessions: small firms hire back workers being laid off by large firms. This hiring could be important for the survival and growth of small firms as 24% of small business owners complain that a lack of qualified workers is a threat to their survival (National Small Business United, 1996). During recessions, the labor market frictions impose less constraints on small firms that have an investment opportunity but lack of workers. These firms take the advantage of a favorable labor market and grow fast during recessions.

In the model firms may have investment opportunities in any period with a positive probability. If a firm does have an investment opportunity, it receives idiosyncratic shocks to the fixed capital adjustment costs. A firm only invests if this fixed cost is relatively low; otherwise, the capital depreciates in the firm. This random capital adjustment cost can generate an investment pattern that is consistent with a well known regularity: investment stays inactive for a few periods when the capital adjustment cost is relatively high, and it spikes when the capital adjustment cost becomes relatively low. The history of investment thus determines the marginal labor productivity in a firm and, therefore, its employment decision. Consequently, the investment opportunity and capital adjustment frictions affect firm sizes.[1]

Introducing labor search frictions disentangles firms' employment dynamics from the dynamics of their capital stock. If the labor market is perfectly competitive, a firm's size of employment will be perfectly correlated with its capital stock. With Walrasian labor market firms' investment decisions are completely determined by exogenous shocks. Search frictions imply that even if two firms have the same history of investment, their employment levels can still be different, depending on the outcome of their labor search and matching. The resulting size differences, in turn, affect investment decisions.

We calibrate the model to United States data and compute the stochastic equilibrium. The model can explain the puzzling fact (2): the procyclical nature of job destruction in small firms. The story is as follows. During booms, when aggregate productivity goes up, the marginal productivity of capital and labor generally increases. Firms invest more and hire more workers. The labor market becomes tighter as the unemployment rate becomes lower. A tighter labor market has asymmetric effects on small and large firms. In the model, small firms are those that have lacked investment opportunities and forgone investment for many periods. These small firms are likely to make lumpy investment once they have a chance and face a low capital adjustment cost because they may not have a good chance in

[1] There are different theories about how firm size is determined. Lucas (1967) proposed a story where investment adjustment costs determine firm sizes.

the next period. A tighter labor market reduces firms' incentives to invest, especially the incentives of small firms. This is because small firms need to increase their employment in a higher proporiton once they make a lumpy investment. Therefore, the profit margin of a small firm's investment is more affected by labor market tightness. A tighter labor market deters marginal investment projects more substantially in small firms.

Because the increase of aggregate productivity and a tighter labor market have asymmetric effects on small and large firms, the rise of investment hazard rate (the probability of investment) increases with size. In the numerical experiment a permanent positive productivity shock of 1% increases the investment hazard rate by 5% in small firms and by 14% in large firms. As investment increases relatively more in large firms, the relative marginal labor productivity in small firms decreases, and workers migrate from small firms to large firms. This is how job destruction in small firms may increase during booms.

These asymmetric effects do not exist if investment is not lumpy. When a firm makes a lumpy investment it expects either to not have an investment opportunity or to have a higher capital adjustment cost in next period. Therefore, it takes into account the future employment opportunities and thus affects its future employment dynamics. If there are no capital adjustment frictions, a firm's investment decision will depend only on its current marginal productivity of capital, which is determined by the current level of employment but not the expected future level of employment. In that case, the current investment of a firm has no effect on the firm's employment dynamics. The employment dynamics will be determined by the aggregate productivity shocks, which are identical to all firms.

The facts this paper seeks to explain were documented by Davis and Haltiwanger (1992) using U.S. manufacturing data. More recently, Moscarini and Postel-Vinay (2008) reported that both the employer to employer flow rate of workers and wages increase rapidly in the late expansion phase of the business cycle. To explain these phenomena, they propose a mechanism by which large firms strategically steal workers from small firms by posting a higher wage in the late expansion. Their model is not a standard business cycle framework since the model covers only economic expansions but not recessions. If there is a negative shock, their model cannot be solved. This paper extends the standard business cycle model and postulates an alternative mechanism by which the interaction between labor search and lumpy capital contributes to the propagation of business cycles and the allocation of workers between small and large firms.

This paper builts on a model by Khan and Thomas (2003) that was designed to capture the empirical fact that individual firms forgo investing during some periods and have dramatic surges in investment during some other periods.[2] Cooper and Haltiwanger (2006) find that a model with non-convex adjustment costs and irreversibility of capital fits prominent features of observed investment behavior at the micro level. Interestingly, Cooper, Haltiwanger and Wills (2006) find a similar discrete adjustment pattern for employment. They use a labor search model with non-convex vacancy posting costs to explain this fact. Their paper abstracts from capital. In this paper, a non-convex capital adjustment cost generates both lumpy capital and lumpy employment adjustments.

The rest of the paper is organized as follows. Section 2 describes the data and facts; section 3 sets up the model; section 4 defines the equilibrium and discusses the model solution and implications; section 5 sketches the computational algorithm; section 6 calibrates the model parameters; and section 7 analyzes the results. Finally, section 8 concludes.

2 Data and Facts

The data used in this paper come from the Business Employment Dynamics (BED) survey (1992-2007). Firm size is defined by the current number of employees. Small firms are firms with 1-249 employees (49.3% employment share).[3] The data set covers the entire private sector, including all the firms covered under state unemployment insurance (UI) programs (which account for 98% employment). The data are measured quarterly.

[2] See Caballero, Engel and Haltiwanger 1995, Doms and Dunne 1998, Caballero and Engel 1999, Cooper and Haltiwanger 2006, and Gourio and Kashyap 2007 for empirical evidence; see Thomas 2002, Khan and Thomas 2008, Bachmann, Caballero and Engel 2006, and Gourio and Kashyap 2007 for theoretical models.

[3] The Small Business Administration (SBA) has defined small businesses in different ways. In the late 1950s, the agency viewed as "small" all industrial establishments with fewer than 250 employees. So the early studies use this definition (e.g. Davis and Haltiwanger 1992). In 1988, reflecting the growing sizes of businesses in the United States, the SBA was defining any firm with 500 or fewer empoyees as small, though the acceptable maximum number of employees might vary by industry group: 500 employees for most manufacturing and mining industries; 100 employees for all wholesale trade industries. A more precise breakdown of the size categories in use by the SBA is: under 20 employees, very small; 20-99, small; 100-499, medium-sized; and over 500, large (SBA, Annual Report, 1988, 19. Also see Blackford 1991).

The data set reports the changes in employment between each quarter's third month. Job creation is the sum of all employment gains at (i) continuous firms expanding their employment, and (ii) "opening" firms reporting positive employment either for the first time or after reporting zero employment in the previous quarter. Job destruction is the sum of all employment losses at (i) continuous firms contracting their employment, and (ii) "closing" firms either disappearing or reporting zero employment after reporting positive employment in the previous quarter. Using this data set, table 2.1 and figure 2.1 exhibit the stylized facts mentioned in the introduction.

Table 1 shows the cross correlations between output and job creation and destruction in small and large firms. (1) Job creation in both small and large firms is positively correlated with output, so it is procyclical. (2) Job destruction in small firms is positively correlated with output, while job destruction in large firms is negatively correlated with output. So job destruction in small firms is procyclical, while job destruction in large firms is countercyclical. (3) The standard deviations of job creation and destruction in large firms are about 2.5 times as large as in large firms.

Table 1 Cyclical Behavior of the U.S. Job Creation and Job Destruction:
In Small and Large Firms
Deviations from Trend, 1992:III-2007:I

		Cross-Correlation of Output with								
Variable	*SD%*	$x(-4)$	$x(-3)$	$x(-2)$	$x(-1)$	x	$x(+1)$	$x(+2)$	$x(+3)$	$x(+4)$
GDP	0.84	0.354	0.510	0.707	0.852	1.0	0.852	0.707	0.510	0.354
C_S	2.51	0.329	0.486	0.611	0.683	0.618	0.527	0.332	0.258	0.052
C_L	6.18	0.275	0.402	0.462	0.483	0.450	0.380	0.255	0.211	−0.024
De_S	2.51	−0.150	−0.099	−0.069	0.016	0.182	0.376	0.490	0.535	0.595
De_L	6.81	−0.244	−0.257	−0.298	−0.246	−0.178	0.054	0.194	0.412	0.547

Note: The variables: C_S (C_L): log of job creation in small firms (large firms); De_S (De_L): log of job destruction in small firms (large firms). The data are quarterly series and expressed as deviations from a Hotric-Prescott filter with smoothing parameter 1600.

Figures 1.a and 1.b visualize the deviations of log job creation and log job destruction in small and large firms from their Hotric-Prescott trend, compared to the deviations of log GDP from its Hotric-Prescott trend. Figure 1.a shows that job creation rate in small firms moves together with job creation rate in large firms, and that job creation rate in both small and large firms increases when GDP growth rate is high. So job creation is procyclical in both small and large firms. Moreover, job creation rate goes up and down by more in large firms, implying that job creation is more volatile in large firms.

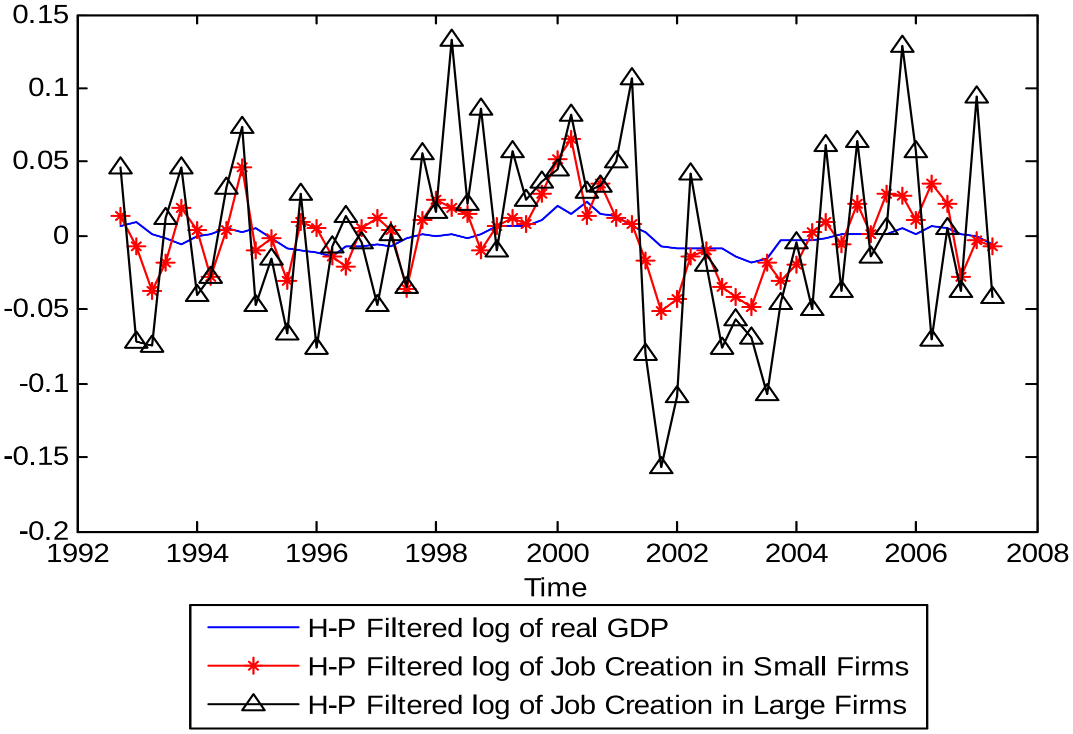


Figure 1.a H-P Filtered Cyclical Component of Job Creation in Small and Large Firms

Figure 1.b shows that, in economic downturns when GDP growth rate is low, job destruction rate first increases in both small and large firms, but job destruction rate in small firms may start to decrease while job destruction rate in large firms is still going up. In economic booms, job destruction rate first decreases in both small and large firms, but job destruction rate in small firms may start to increase while job destruction rate in large firms is still going down. So job destruction in small firms may be procyclical, while job destruction in large firms is countercyclical. Moreover, job destruction rate goes up and down by more in large firms, implying that job destruction is more volatile in large firms.

The different behavior in small firms versus large firms suggests two effects after an aggregate productivity shock: a productivity effect and a composition effect. While both small and large firms respond to an aggregate productivity shock in the same direction at the beginning, the competition between small and large firms changes and the composition effects become dominant as the shock propagates.

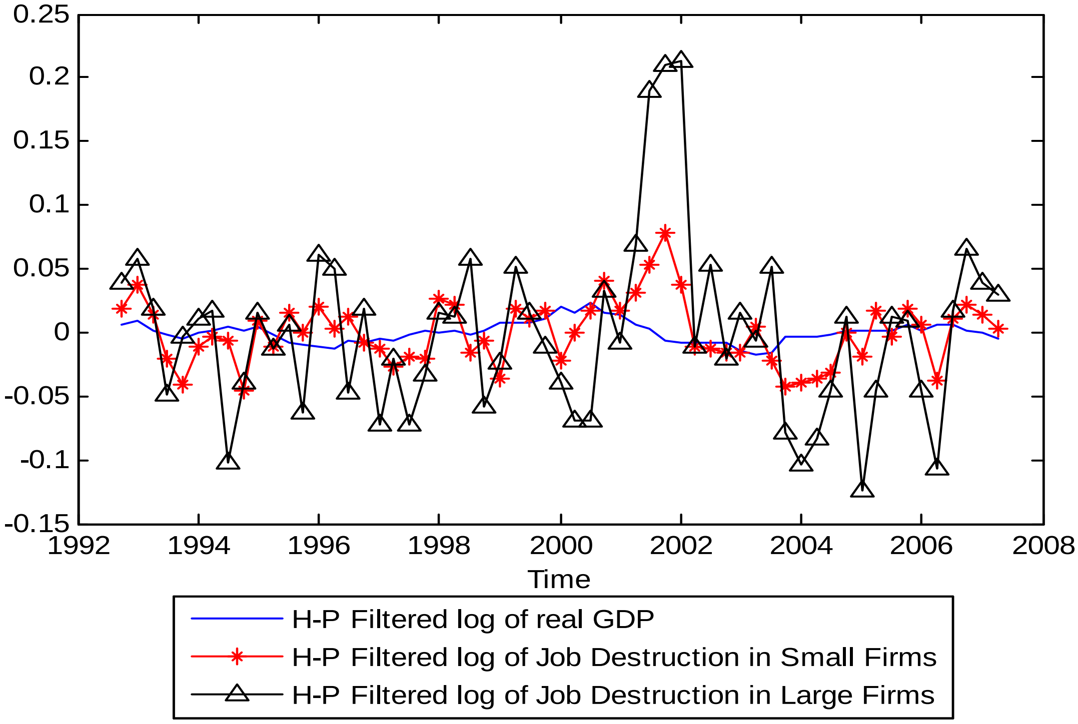


Figure 1.b H-P Filtered Cyclical Component of Job Destruction in Small and Large Firms

To complete the analysis of the data, figure 2 shows the first moments of job creation and job destruction. The average job creation rate (job creation divided by the total number of employees) and job destruction rate (job destruction divided by the total number of employees) are higher in small firms.

Figure 2 Firm Sizes and Job Creation Rate and Destruction Rate

The above facts are described by data at firm level. Using the unpublished data at establishment level from the BED survey, Moscarini and Postel-Vinay (2008) find that the pattern of establishment size dynamics (employment dynamics) over the last two business cycles closely resembles that of firm size dynamics. Part of this resemblance is due to the fact that most (small) firms are mono-establishment, while large establishments tend to be part of large firms, as shown in Table 2.

Table 2 Firm Sizes and Establishments

Firm size category	Average number of establishments	Mean establishment size
all	1.26	15.58
$1-4$	1.00	2.10
$5-9$	1.01	6.49
$10-19$	1.05	12.75
$20-99$	1.32	29.80
$100-499$	3.82	50.71
500 and up	61.98	53.46

Source: Moscarini and Postel-Vinay (2008), according to County Business Pattern data set

In general, employment dynamics show co-movements in different sectors (Moscarini and Postel-Vinay 2008). The manufacturing sector is different since its establishments are larger on average. Nevertheless, as shown in Davis, Haltiwanger and Schuch (1996) using U.S.

manufacturing industry data from 1972 to 1986, during recessions, large establishments experience sharply higher job destruction rates, so their contribution to the job destruction rises. Although they did not explicitly point out that small establishments have procyclical job destruction, it is implied by table 3 (quoted below from their book) since job creation and job destruction are positively correlated in small establishments.

Table 3 Correlation between Job creation and Destruction by Establishment Sizes

Establishment size category	Correlation of job creation and destruction
$0 - 19$	0.45
$20 - 49$	0.11
$50 - 99$	-0.15
$100 - 249$	-0.47
$250 - 499$	-0.47
$500 - 999$	-0.44
1000 and up	-0.43

Source: Davis, et. al. (1996) according to the Annual Survey of Manufactures between 1972 and 1986.

During expansion years, the percentage of job destruction from establishment shutdown increases by more in large firms than in small firms. This rules out the hypothesis that the main reason for the increased job destruction during expansion years is the increased entry and exit.

Table 4 Job destruction from shutdown in establishments with different sizes
in U.S. manufacturing industry

	Recession years	Expansion years	Change
fewer than 50	30%	34%	113%
50-249	25%	30%	120%
250-999	16%	22%	138%
1000 or more	8%	14%	175%

Source: Schuh and Triest (1998) according to the Annual Survey of Manufactures between 1972 and 1986

For easy exposition, the production unit in the model below will be an establishment. We will also impose the strong assumption that large firms are composed of large establishments while small firms are composed solely of small establishments. With this assumption, the model's predictions apply to both firm sizes and establishment sizes.

3 The Model

3.1 Preferences

The economy is populated by a unit measure of identical households. Households face the standard consumption-saving problem. In addition, they face different opportunities for exchanging labor services. In particular, individuals either have a job opportunity or not, and job opportunities come and go at random. Having a job opportunity means being matched with an establishment, and having the opportunity to negotiate a labor contract that stipulates the terms by which labor services are exchanged for wages. This household structure improves the tractability of the model in an environment with search frictions (see Shi 1997).

A typical household has preferences represented by a utility function of the following form:

$$E_0 \sum_{t=0}^{\infty} \beta^t [U(c_t) - A \; N_t]$$

where c_t denotes consumption, N_t denotes the fraction of individuals being employed, and $0 < \beta < 1$ is the discount factor. The function U is increasing and concave in c_t. A is the marginal disutility of working. This utility function can be interpreted as a reduced-form of the indivisible labor model in Hansen (1985) and Rogerson (1988).[4]

3.2 Production Technology

Output, which can be consumed or invested, is produced by a large number of establishments with the following production function:

$$y = zk^a n^b, \tag{1}$$

where z is aggregate productivity, k is capital, n is labor, $a > 0$, $b > 0$, and $a + b < 1$. Aggregate productivity is a stochastic variable common to all establishments, and follows a Markov

[4] In Hansen (1985) the utility function takes the form

$$N(\log c + A \log(1 - h)) + (1 - N)(\log c + A \log 1),$$

where h is working hours. By rearranging it and omitting the constant terms we can obtain a momentary utility function of the form $\log(c) - A \log(1 - h)N$. h is assumed to be constant in this paper.

process with a finite support and a transition matrix Π described by

$$\Pr(\ z' = z_j \mid z = z_i\ ) = \pi_{ij} \geq 0,$$

and $\sum_{j=1}^{J} \pi_{ij} = 1$ for each $i = 1, ...J$. Given that all the establishments have access to the same production technology, ex ante, they are identical.

3.3 Capital Adjustment

After current production takes place, each establishment has an opportunity to invest with probability ψ.[5] This opportunity enables establishments to make a positive investment with a fixed cost of capital adjustment $\xi \in (0, \bar{\xi})$ drawn from a time-invariant distribution $G(\xi)$ common to all establishments. Within a period, the capital adjustment cost is fixed at the establishment level and is independent of the level of capital adjustment. At any point in time, given the differences in investment opportunities and in the magnitudes of fixed adjustment costs across establishments, some establishments will adjust their capital stocks while others will not. As a result, establishments possess different capital stocks even in the absence of idiosyncratic productivity shocks.

An establishment's capital evolves over time according to

$$k' = \begin{cases} (1-\delta)k + i & \text{with probability } \psi \\ (1-\delta)k & \text{with probability } 1-\psi \end{cases} \tag{2}$$

where $i \geq 0$ is the establishment's current investment and $\delta \in (0,1)$ is the rate of capital depreciation.

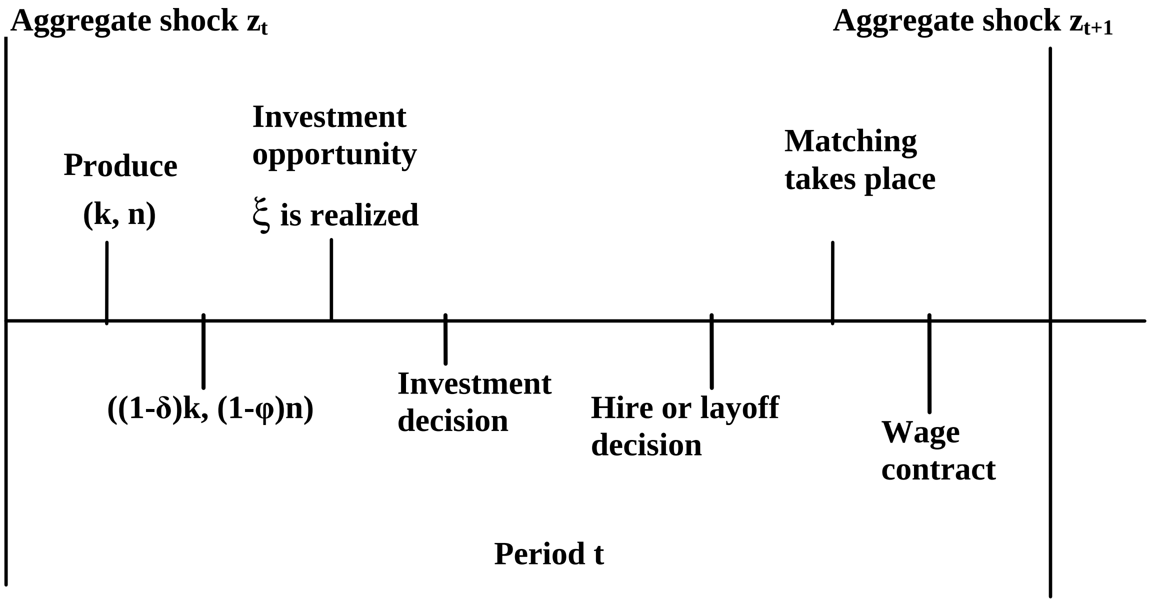
3.4 Labor Search

Workers and producers are brought together through a search process. A worker who is matched with a producer earns a wage specified by a state-contingent contract that depends on the establishment's size and marginal labor productivity. Workers are bound by the contract until they are fired or hit by an exogenous job separation shock. In order to have a clear perspective, we first describe the order of events within a period.

[5] The main reason for allowing this random opportunity of investment is to reconcile the differences in frequencies of investment and employment adjustments. The usual frequency of investment is one year, while the employment adjustment happens every month. So ψ is taken as 1/12 in this paper since a typical period is one month here.

3.4.1 Time Line

The timing of events within a period is described as follows: (1) the aggregate shock is realized; (2) establishments produce with the capital and the workers inherited from the past; (3) investment opportunity shocks are realized and establishments with opportunities to invest draw capital adjustment costs from the distribution $G(\xi)$ and make capital adjustment decisions; (4) establishments decide how many workers they would like for the next period and either fire workers or post vacancies; (5) unemployed workers and vacancies are matched randomly and a state-contingent contract is signed; and (6) the period concludes and the next period starts with the same order of events.



Note that matching is completed before the next period's aggregate productivity shock is realized, and no firing is allowed between the time matching takes place and the next production period.

3.4.2 Matching Mechanism

Firms are allowed to post multiple vacancies and every position is matched randomly. The aggregate matching function is $M(\tilde{\mathbf{v}}, (1-N))$, where $\tilde{\mathbf{v}}$ is the aggregate number of vacancies, and $1-N$ is the aggregate unemployment. Establishments can recruit by posting vacancies,

v , with a vacancy cost $e > 0$. The proportion of vacancies that are filled, $x \in [0,1]$, is random and distributed according to $f(x)$, which is related to the aggregate matching function as follows:

$$f(x) = C_v^{xv} h^{xv} (1-h)^{v-xv}.$$

Here, $h = M(\tilde{\mathbf{v}}, (1-N)) \ / \ \tilde{\mathbf{v}}$ represents the average vacancy-filling rate, $C_v^{xv} \equiv v!/[(xv)! \ (v-xv)!]$ represents the number of ways that xv out of v vacancies can be filled, and v is the number of vacancies posted by an establishment.[6] Every establishment takes $f(x)$ as given. The CDF of x is denoted by $F(x)$.

3.4.3 The Wage Contract

The wage contract is signed before the establishment's state is realized. Once a contract is signed the worker cannot leave the establishment except when being fired or being hit by an exogenous separation shock. The wage is contingent on the marginal productivity of labor of the establishment realized every period according to the following rules: (1) in the case where the marginal productivity of labor is not greater than the disutility of working, the wage is equal to the disutility of working in terms of good, so $w = A/p$, where p is the utility value of current goods; (2) in the case where the marginal productivity of labor is greater than the disutility of working, $w = (MP_L + A/p)/2$, where MP_L is the marginal productivity of labor. The wage is updated every period, and it is identical for new and existing workers.

3.5 Distribution of Establishments and Decision Rules

The aggregate state variables at the beginning of each period are the aggregate productivity shock z and the distribution of establishments $\bar{\boldsymbol{\mu}}$ described by a probability measure $\mu(k,n)$ over capital and employment, which is defined on the product space $S = R_+ \times R_+$. The distribution of establishments evolves over time according to a mapping Γ from the current aggregate state to a new one: $\bar{\boldsymbol{\mu}}' = \Gamma(z, \bar{\boldsymbol{\mu}})$. This mapping is endogenous and determined below.

Let $V^0(k,n;z_i,\bar{\boldsymbol{\mu}})$ denote the expected value of an establishment at the beginning of a period, prior to the realization of its adjustment cost but after the determination of

[6]Note that xv is not an integer in this paper. The computation involves approximation.

$(k, n; z_i, \bar{\boldsymbol{\mu}})$. Let $\check{V}^1(k, n; z_i, \bar{\boldsymbol{\mu}})$ denote the expected discounted value of an establishment that enters the period with (k, n), and has no opportunity to invest. Let $V^1(k, n, \xi; z_i, \bar{\boldsymbol{\mu}})$ denote the expected discounted value of an establishment that enters the period with (k, n), has an opportunity to invest, and draws an adjustment cost ξ.

Consider an establishment that has drawn the investment cost ξ and has decided to invest. The expected future value of the establishment, net of investing and hiring costs, is

$$\tilde{\Delta}_I = \max_{k'_I} \left\{ -\xi - i + \max_{v, f^i} \left[-ev + \sum_{j=1}^{J} \pi_{ij} d_j(z, \bar{\boldsymbol{\mu}}) \int_0^1 V^0(k'_I, n'; z_j, \bar{\boldsymbol{\mu}}') F(dx) \right] \right\}. \tag{3}$$

Here, e is the vacancy posting cost. If the establishment invests, it chooses an optimal level of k'_I. The investment is given by $i = k'_I - (1 - \delta)k$. n is the number of workers in the current period. The establishment chooses to either post vacancies v or to fire workers f^i. The number of workers in the next period also depends on the realization of the individual matching rate x. The evolution of employment for an establishment is

$$n' = (1 - \varphi)n + vx - f^i \ , \tag{4}$$

where **either** v **or** f^i is positive, or both are equal to zero. $d_j(z_i, \bar{\boldsymbol{\mu}})$ is the discount factor applied by establishments to their next period expected value if aggregate productivity at that time is z_j and current productivity is z_i. (Except where necessary for clarity, we suppress the indices for current aggregate productivity below.) Suppose, instead, that this establishment chooses not to invest. Then, the net expected future value would be

$$\tilde{\Delta}_{no} = \max_{v, f^i} \left[-ev + \sum_{j=1}^{J} \pi_{ij} d_j(z, \bar{\boldsymbol{\mu}}) \int_0^1 V^0(\ (1 - \delta)k, n'; z_j, \bar{\boldsymbol{\mu}}') F(dx) \right]. \tag{5}$$

The value functions $V^0(k, n; z_i, \bar{\boldsymbol{\mu}})$, $\check{V}^1(k, n; z_i, \bar{\boldsymbol{\mu}})$, and $V^1(k, n, \xi; z_i, \bar{\boldsymbol{\mu}})$ satisfy the following Bellman equations:

$$V^0(k, n; z_i, \bar{\boldsymbol{\mu}}) \equiv (1 - \psi)\check{V}^1(k, n; z_i, \bar{\boldsymbol{\mu}}) + \psi \int_0^{\bar{\xi}} V^1(k, n, \xi; z_i, \bar{\boldsymbol{\mu}}) G(d\xi), \tag{6}$$

$$\check{V}^1(k, n; z_i, \bar{\boldsymbol{\mu}}) = \ zF(k, n) - wn + \tilde{\Delta}_{no}, \tag{7}$$

and

$$V^1(k, n, \xi; z_i, \bar{\boldsymbol{\mu}}) = \ zF(k, n) - wn + \max(\tilde{\Delta}_I, \tilde{\Delta}_{no}). \tag{8}$$

Establishments start producing right after the aggregate shock is realized. After production, an establishment with an opportunity to invest chooses the optimal investment level. Those with positive investments pay the capital adjustment costs. However, if establishments do not invest, this cost is avoided as shown in $\tilde{\Delta}_{no}$. Next, establishments make hiring or firing decisions. They either post vacancies with cost e or fire workers without incurring any costs, depending on the expected future aggregate conditions.

Let $k^f(\ k, n, \xi; z, \bar{\boldsymbol{\mu}})$ denote the choice of capital in the next period by establishments of type (k, n) with adjustment cost ξ. Let $v(k, n; z, \bar{\boldsymbol{\mu}})$ denote the choice of vacancies and $f^i(k, n; z, \bar{\boldsymbol{\mu}})$ denote the number of layoffs by all type (k, n) establishments. The aggregate employment for the next period is

$$N' = \int_S \int_0^1 \left[(1-\varphi)n(k, n; z, \bar{\boldsymbol{\mu}}) + v(k, n; z, \bar{\boldsymbol{\mu}})\ x - f^i(k, n; z, \bar{\boldsymbol{\mu}})\right] dF(x)\mu(d\ [k \times n]\ ). \tag{9}$$

The aggregate number of vacancies is $\tilde{\mathbf{v}} = \int_S\ v(k, n; z, \bar{\boldsymbol{\mu}})\ \mu(d\ [k \times n]\ )$ and the aggregate number of layoffs is $\tilde{f}^i = \int_S\ f^i(k, n; z, \bar{\boldsymbol{\mu}})\ \mu(d\ [k \times n]\ )$.

3.6 The Household's Problem

Each household holds shares of the establishments, which are denoted by a measure $\lambda(k, n)$. The employment N is taken as a state variable. The household chooses current consumption, c, and the number of new shares $\lambda'(k, n)$ to purchase at price $\rho(k, n; z, \bar{\boldsymbol{\mu}})$. Denote $\bar{\lambda}$ as a vector of the shares over (k, n) and $\bar{\rho}$ as a vector of the prices of the shares. The household's utility maximization problem is described by the Bellman equation below:

$$W(\bar{\lambda}, N; z, \bar{\boldsymbol{\mu}}) = \max_{\{c, \bar{\lambda}'\}} \{U(c) - AN + \beta \sum_{j=1}^{J} \pi_{ij} W(\bar{\lambda}', N'; z_j, \bar{\boldsymbol{\mu}}')\}. \tag{10}$$

The budget constraint is:

$$\begin{aligned} & c + \int_S \rho(k, n; z, \bar{\boldsymbol{\mu}})\lambda'(d[k \times n]) \\ \leq & \int_S w(k, n; z, \bar{\boldsymbol{\mu}})n(k, n;\ z, \bar{\boldsymbol{\mu}})\mu(d[k \times n]) + \int_S V^0(k, n; z, \bar{\boldsymbol{\mu}})\lambda(d[k \times n]). \end{aligned} \tag{11}$$

Letting $C(\bar{\lambda}, N; z, \bar{\boldsymbol{\mu}})$ be the policy function describing the optimal choice of current consumption, and $\Lambda(\bar{\lambda}, N; z, \bar{\boldsymbol{\mu}})$ be the policy function describing the optimal choice of the shares that the household purchases of the establishments.

4 The Equilibrium

4.1 Definition of the Recursive Equilibrium

A recursive equilibrium is consists of a set of value functions $(W, V^0, V^1, \check{V}^1)$; a set of policy functions for the household C and Λ; a set of policy functions for the establishments $k^f, v,\ f^i$; a set of prices p and $\bar{\rho}$; a set of average matching rate h, and a set of distribution measures $\bar{\lambda}$ and $\bar{\boldsymbol{\mu}}$ such that:

1. Given the prices $p(z, \bar{\boldsymbol{\mu}})$ and the aggregate matching rate h, V^0, V^1 and $\check{V}^1$ satisfies (3) - (8) and $(k^f, v,\ f^i)$ are the associated policy functions for the establishments;

2. Given the prices $p(z, \bar{\boldsymbol{\mu}})$ and $\bar{\rho}$, W satisfies (10) and (C, Λ) are the associated policy functions for the households;

3. The law of motions of aggregate employment and capital stock are consistent with the individual establishments' behavior:

$$N' = \int_S \int_0^1 \Big[(1-\varphi)n(k,n;z,\bar{\boldsymbol{\mu}}) + v(k,n;z,\bar{\boldsymbol{\mu}})\ x - f^i(k,n;z,\bar{\boldsymbol{\mu}})\Big]\, dF(x)\mu(d[k \times n]); \tag{12}$$

$$K' = \int_S [(1-\delta)k(k,n;z,\bar{\boldsymbol{\mu}}) + i(k,n;z,\bar{\boldsymbol{\mu}})\ ]\ \mu(d[k \times n]); \tag{13}$$

4. The law of motion of $\bar{\boldsymbol{\mu}}$:

$$\bar{\boldsymbol{\mu}}' = \Gamma(z, \bar{\boldsymbol{\mu}});$$

5. The share market clears, i.e. $\Lambda(\bar{\lambda}, N; z, \bar{\boldsymbol{\mu}}) = \bar{\mu}$;

6. The goods market clears:

$$\begin{aligned} C(\bar{\lambda}, N; z, \bar{\boldsymbol{\mu}}) &= \int_S \{zF(k, n'(k,n;z,\bar{\boldsymbol{\mu}}))\mu(d[k \times n]) \\ &\quad -\psi \int_S \int_0^{\bar{\xi}} (k^f(k,n,\xi;z,\bar{\boldsymbol{\mu}}) - (1-\delta)k)G(d\xi)]\mu(d[k \times n]) \\ &\quad -\psi \int_S D\ [\alpha(k,n)]\mu(d[k \times n]) - \tilde{\mathbf{v}}\ e, \end{aligned} \tag{14}$$

where

$$D\ [\alpha(k,n)] = \int_0^{G^{-1}(\ \alpha(k,\ n)\ )} \xi G(d\xi). \tag{15}$$

$D\ [\alpha(k,n)]$ is the average value of adjustment costs of all type (k,n) establishments that invest in capital. Letting $\hat{\xi}$ be the highest adjustment cost such that the type (k,n) establishments undertake positive investment and $\alpha(k,n)$ be the fraction of type (k,n) establishments that invest in capital, then $G(\hat{\xi}) = \alpha(k,n)$. An establishment chooses to invest if it draws $\xi \in (0,\hat{\xi})$, and not to invest if it draws $\xi > \hat{\xi}$.

4.2 Model Solution and Discussion

The equilibrium is computed by solving a single Bellman equation that combines establishments' profit maximization problem with the utility maximizing conditions from the household's problem. Let $p(z,\bar{\boldsymbol{\mu}})$ be the utility value of current goods (the multiplier for the budget constraint in the household maximization problem). The first order condition in the household problem gives

$$p(z,\bar{\boldsymbol{\mu}}) = U'(c). \tag{16}$$

The discounting factor is defined as $d_j(z,\bar{\boldsymbol{\mu}}) \equiv \frac{\beta U'(c')}{U'(c)} = \frac{\beta p(z_j,\bar{\boldsymbol{\mu}}')}{p(z,\bar{\boldsymbol{\mu}})}$.

Establishments use $p(z,\bar{\boldsymbol{\mu}})$ to evaluate current output. A reformulation of equations (3) - (8) yields an equivalent description of the establishments' dynamic problem. Following Khan and Thomas (2003), rather than subtracting investment from current profits, we let the value of non-depreciated capital be included in the current profits, and let the establishment "repurchase" its capital stock each period. This is done only for expositional convenience. Suppressing the arguments of the price functions, the value function of an establishment with no investment opportunity becomes

$$\check{V}^1(k,n;z_i,\bar{\boldsymbol{\mu}}) = [\ zF(k,n) - wn + (1-\delta)k]\ p\ +\ \Delta_{no}\ , \tag{17}$$

and the value function of an establishment with investment opportunity and with a draw ξ becomes

$$V^1(k,n,\xi;\ z_i,\bar{\boldsymbol{\mu}}) = [\ zF(k,n) - wn + (1-\delta)k]\ p +\ \max(\Delta_I,\Delta_{no}). \tag{18}$$

In equation (18), Δ_I is the net value of achieving the target capital, while Δ_{no} is the continuation value of the establishment if it does not invest in capital. Δ_I and Δ_{no} are given below:

$$\Delta_I = \max_{k'_I}\left\{-\xi p - k'_I p + \max_{v,f^i}\left[-evp + \beta\sum_{j=1}^{J}\pi_{ij}\int_0^1 V^0(k'_I,n';z_j,\bar{\boldsymbol{\mu}}')F(dx)\right]\right\}, \tag{19}$$

and

$$\Delta_{no} = -(1-\delta)kp + \max_{v,f^i}\left[-evp + \beta\sum_{j=1}^{J}\pi_{ij}\int_0^1 V^0((1-\delta)k, n'; z_j, \bar{\boldsymbol{\mu}}')F(dx)\right]. \tag{20}$$

Here, k_I' is the next period's capital level if the establishment chooses to invest. The employment evolves according to (4), and $V^0(k, n; z_i, \bar{\boldsymbol{\mu}})$ is defined in (6).

Now we examine the establishments' decisions. After the current period production takes place, an establishment with investment opportunity draws ξ. If this ξ is relatively low, the establishment undertakes investments, and the optimal capital stock $\hat{k}_I'(n; z_j, \bar{\boldsymbol{\mu}})$ solves the right side of (19). Note that the optimal level of capital stock next period $\hat{k}_I'(n; z_j, \bar{\boldsymbol{\mu}})$ is independent of the current level of capital stock k and capital adjustment cost ξ. This is because both the marginal cost of purchasing new capital, p, and the marginal benefit of purchasing new capital do not depend on k and ξ. Denote $X = \beta\int_0^1\sum_{j=1}^{J}\pi_{ij}V^0(k_I', n'; z_j, \bar{\boldsymbol{\mu}}')F(dx)$ as the expected future value for an easy exposition. It is clear that the marginal increase in the expected future value of the establishments with respect to k_I', $\left.\frac{\partial X}{\partial k_I'}\right|_{\hat{k}_I'} = \left[\beta\int_0^1\sum_{j=1}^{J}\pi_{ij}\frac{\partial\ V^0\ (k_I',\ n';\ z_j,\ \bar{\mu}')}{\partial\ k_I'}F(dx)\right]_{\hat{k}_I'}$, do not depend on k and ξ.

As a result, all establishments with positive investments and equal employment stocks n will choose a common level of capital for the next period. Because the optimal level of capital in the next period is independent of the current capital level, the net value of achieving the optimal capital level, $\Delta_I(\xi, n; z, \bar{\boldsymbol{\mu}})$, is also independent of current capital. However, both the optimal level of capital stock, $\hat{k}_I'$, and the level of Δ_I depend on the current level of employment in the establishments through (4). This is an important implication and is restated in the following proposition. (The proof is straight forward and therefore we omit it.)

Proposition 1 *With labor search frictions and fixed capital adjustment costs, establishments' optimal levels of capital stock conditional on making positive investment are independent of the current individual capital stocks, but they depend on the establishments' sizes measured by their current employment.*

Labor market search is important for the non-trivial dependence of the optimal capital stock on an establishment's current employment. If there were no search frictions in the labor market, the model would predict that all the establishments would choose the same optimal capital stock, and one level of capital stock would be associated with one level of employment, as in Khan and Thomas (2003). The unrealistic prediction that one level of capital stock is always associated with one level of employment is avoided in the current setting by the presence of search frictions in the labor market. With random matching, the establishments that have the same history of capital adjustment cost shocks and labor market matching outcomes and desire to have the same level of capital stock will still end up with different levels of employment in the next period. If the current level of employment is relatively low and the establishment needs to hire more workers to implement the new investment, there will be a wider distribution of possible states of employment in the next period. The marginal benefit of investment in establishments with fewer workers will be relatively lower given that the expected future value is concave in employment. Consequently, a larger firm size has positive effect on investment.

Now consider the establishments that do not undertake investments. Since these establishments do not invest and their capital stocks depreciate with a fixed rate, their capital stocks are reduced. The continuation value for such an establishment is Δ_{no}, which is positively related to the current capital stock. Again, all the establishments with type (k,n) will choose the same level of v or f^i, but the realized levels of the next period's employment will depend on the realization of the individual job filling rate.

From (19) and (20) it is now clear that an establishment will undertake positive investment only if the net value of achieving the target capital, $\Delta_I(\xi,n;z,\bar{\boldsymbol{\mu}})$, exceeds its continuation value under non-adjustment $\Delta_{no}(k,n;z,\bar{\boldsymbol{\mu}})$. It follows immediately that an establishment of type (k,n) will undertake capital adjustments if its fixed adjustment cost, ξ, falls below a threshold value, $\tilde{\xi}(k,n;z,\bar{\boldsymbol{\mu}})$, which depends on (k,n). At $\xi=\tilde{\xi}(k,n;z,\bar{\boldsymbol{\mu}})$, an establishment is indifferent between adjusting its capital stock and not adjusting its capital stock. That is,

$$\Delta_i(\tilde{\xi},n;z,\bar{\boldsymbol{\mu}})=\Delta_{no}(k,n;z,\bar{\boldsymbol{\mu}}).$$

Define the threshold value of the capital adjustment cost as

$$\hat{\xi}(k,n;z,\bar{\boldsymbol{\mu}})\equiv\min\left\{\bar{\xi},\ \max\{0,\tilde{\xi}(k,n;z,\bar{\boldsymbol{\mu}})\}\right\}.$$

Establishments with adjustment costs at or below $\hat{\xi}(k, n; z, \bar{\boldsymbol{\mu}})$ will adjust their capital stock. This threshold value determines the investment hazard rate.

Another implication of introducing labor search is that the investment hazard rate now is not only determined by the capital stock, but also by the employment stock. In Khan and Thomas (2003) the investment hazard rate strictly decreases with the capital stock, which implies that small firms always have higher investment hazard rates (Note that a lower level of capital stock coincides with a lower level of employment). This is not true in this paper. Large establishments (possibly with a low level of capital stock) may have a higher investment hazard rate in a variety of cases. Most obviously, for example, among establishments with identical levels of capital stock the investment hazard rate increases with size (current employment). Given that capital and labor are complementary in production and that it takes time to hire new workers from the frictional labor market, the higher employment level means a higher expected marginal productivity of capital. Moreover, large establishments need less new workers to work with the newly invested capital, which means that a small number of vacancies v is posted. The less costs resulting from the recruiting friction and the smaller vacancy posting costs epv lead to a higher investment hazard rate in larger establishments.

Substitute n' from (4) into the expected future value X gives

$$X = \beta \int_0^1 \sum_{j=1}^{J} \pi_{ij} V^0(k'_I, (1-\varphi)n + vx; z_j, \bar{\boldsymbol{\mu}}')F(dx), \tag{21}$$

if $v \geq 0$. It is obvious that the future value X depends on the current employment n, and the distribution of the individual vacancy-filling rate x. The distribution $F(x)$ is influenced by the average vacancy-filling rate. Since capital and labor are complementary, $\frac{\partial^2 X}{\partial k'_I \partial n}\Big|_{\hat{k}'_I} > 0$, the optimal amount of investment increases in current n. Moreover, the larger the n, the less the vacancies v needed to post, and the smaller the impact of the labor market tightness on X. For large establishments the risk of investment resulting from uncertainty of recruiting is softened by the large current n. Note that the curvature of the production function and, therefore, of the value function V^0 can be important for the quantitative impact of labor search friction and labor market tightness on establishments' investment decisions.

5 Computational Algorithm

In the presence of aggregate uncertainties, establishments need to form rational expectations about the future values induced by their current behavior. To identify the expectation rules that are consistent with rational expectations, we use a guess-and-verify method.

The main computational difficulty of dynamic heterogeneous establishment models is that in order to predict prices, consumers need to keep track of the evolution of the establishment distribution. In other words, the distribution of establishments is one of the aggregate state variables, which means the state space has infinite dimensions. To deal with the problem of a large dimensional state space, we use a small number of moments to approximate the distribution functions as in Krusell and Smith (1998) and, in a context similar to the current paper, Khan and Thomas (2003).

Another problem is that most of the constraints in the maximization problems are nonlinear. Following Khan and Thomas (2003) we solve nonlinearly for V^0 across a multi-dimensional grid of points, using cubic splines to interpolate function values at other points. Johnson et. al. (1993) has shown that this type of multivariate spline approximation is more efficient than multilinear grid approximation.

In a main loop, we guess and verify the functional forms that predict the current equilibrium price, p, the current aggregate vacancy, $\tilde{\mathbf{v}}$, and next period's proxy endogenous state, m'. we denote these functional forms by $p = \hat{p}(z, m; \chi_l^p)$, $\tilde{\mathbf{v}} = \hat{v}(z, m; \chi_l^v)$ and $m' = \hat{\Gamma}(z, m; \chi_l^m)$, where m is a vector of the moments of the distribution of capital stock and employment across establishments, χ_l^p, χ_l^v and χ_l^m are parameters that are determined repeatedly using a procedure explained below, and l indexes these iterations. Every iteration in the main loop contains the following two steps: the inner loop and the outer loop. Every iteration is started with an initial guess of $(\chi_l^p, \chi_l^v, \chi_l^m)$.

1) The inner loop: the first step involves repeated application of the contraction mapping implied by (17)-(20), (4), and (6), given the price (16) and the matching function (10), to solve for V^0. we use $(\chi_l^p, \chi_l^v, \chi_l^m)$, having replaced $\bar{\boldsymbol{\mu}}$ with m and Γ with $\hat{\Gamma}$ in (17)-(20), (4) and (6), to predict the next period's moments of the distribution of capital as well as employment and the current period's equilibrium price and aggregate vacancy. Using the

aggregate levels of employment and vacancies, we can calculate the aggregate matching rate. Given these aggregates, we can solve for V^0 at each point on a grid of values for $(k; n; z; m)$ by iterating over establishments' problems.

2) The outer loop: the second step simulates the economy for T periods. The simulated data are used to estimate the expectation parameters $(\chi_l^p, \chi_l^v, \chi_l^m)$. At the beginning of any period, $t = 1, 2, ...T$, the actual distribution of establishments over capital k and employment n, $\bar{\boldsymbol{\mu}}_t$, is given. we calculate the first moments m directly from the actual distribution $\bar{\boldsymbol{\mu}}_t$. Then we use the approximated mapping $\hat{\Gamma}$ to specify expectations of m'. This procedure determines the expected future value $\beta \int_0^1 \sum_{j=1}^{J} \pi_{ij} V^0(k', n'; z_j, \bar{\boldsymbol{\mu}}')F(dx)$ for any establishment with (k', n'), given V^0 obtained from the first step. After specifying the expectation rules for establishments, we proceed to find the equilibrium price and matching rate: (i) we guess a pair of price and matching rate, $(\tilde{p}\ ,\ \tilde{M})$; (ii) given this pair of price and matching rate, we solve establishments problems to find k^f, v, and f^i using (4), (6) and (17)-(20), and we aggregate these variables; (iii) the aggregate level of employment is given by (13); (iv) the implied price is obtained from (16), and the implied matching rate can be computed given the aggregate level of vacancies $\tilde{\mathbf{v}}$ and the aggregate unemployment $\tilde{\mathbf{u}}$; (v) we check whether the implied price and matching rate converge to the initial guess $(\tilde{p}, \tilde{M})$: if the price and matching rate converge, we calculate the distribution of establishments in the next period, $\bar{\boldsymbol{\mu}}_{t+1}$; if the price and matching rate do not converge, we update the guess for $\tilde{p}$ and $\tilde{M}$ and return to step (i). After the completion of the $T-$periods simulation, the resulting data $(p_t, \tilde{\mathbf{v}}_t, m_t)_{t=1}^T$ are used to re-estimate $(\chi_l^p, \chi_l^v, \chi_l^m)$ using OLS. The estimated $(\chi_l^p, \chi_l^v, \chi_l^m)$ is used in the next iteration.

To sum up, first, we find the value functions of establishments V^0 given a guess of the expectation parameters $(\chi_l^p, \chi_l^v, \chi_l^m)$; second, given V^0, we simulate the model for T periods and obtain simulated data $(p_t, \tilde{\mathbf{v}}_t, m_t)_{t=1}^T$ to estimate the parameters $(\chi_l^p, \chi_l^v, \chi_l^m)$. We iterate these two steps until the parameters $(\chi_l^p, \chi_l^v, \chi_l^m)$ converge. These converged parameters govern the equilibrium expectation rules. Given these parameters, we can simulate the model to obtain data that could be used for analyses.

6 Parameterization

In order to compute the model, we specify the functional forms for U, M, and G. Following the literature, we use an isoelastic utility function for consumption, $U(C) = \frac{C^{1-\theta}}{1-\theta}$, and a standard Cobb-Douglas matching function with bounds, $M(\tilde{\mathbf{v}}, \tilde{\mathbf{u}}) = \min\{\tilde{\mathbf{v}}, \tilde{\mathbf{u}}, \kappa\tilde{\mathbf{v}}^{\gamma}\tilde{\mathbf{u}}^{1-\gamma}\}$, where $\kappa > 0$, and $\gamma \leq 1$. Without loss of generality, we let the capital adjustment cost have a Beta distribution with shape parameters β_p and β_q. The uniform distribution is a special case of the Beta distribution with $\beta_p = 1$ and $\beta_q = 1$. Since the domain of a Beta distribution is $[0, 1]$, we normalize the capital adjustment cost shock $\xi \in [0, \bar{\xi}]$ by $\bar{\xi}$, so that $\xi/\bar{\xi}$ is distributed according to the Beta distribution. Denote the probability distribution function (PDF) as $g(\xi)$, so $g(\xi) = \frac{1}{B(\beta_p, \beta_q)}\xi^{\beta_p - 1}(1-\xi)^{\beta_q - 1}$, where $B\,()$ is the Beta function, $B(\beta_p, \beta_q) = \int_0^1 \xi^{\beta_p - 1}(1-\xi)^{\beta_q - 1} d\xi$.

The rest of this section describes the observations in the U.S. economy, which are used to calibrate the parameters of the model. The parameters to be calibrated are the discount factor β, the coefficient of risk aversion θ, the marginal disutility of working A, the capital and labor shares in the production function a and b, the capital depreciation rate δ, the capital adjustment cost upper bound $\bar{\xi}$, the distributional parameters β_p and β_q, the labor matching function technology parameters κ and γ, the vacancy posting cost e, the exogenous job separation rate φ, and the parameters governing the aggregate productivity shocks. We choose the model time period to be one month to accommodate for the relatively short average durations of unemployment and vacancies in the U.S. economy. Since the average durations of investment is one year, the investment opportunity ψ is set to $1/12$. Calibrating to an annual interest rate of 4 percent, which is a standard value in the macro literature, requires a monthly discount factor β equal to 0.996.

Since the production unit is interpreted as an establishment, we follow Veracierto (2008) in determining the components of the empirical counterparts of variables. The capital components in this paper do not include land, residential structures, and consumer durable goods. The empirical counterpart for investment is associated in the National Income and Product Accounts (NIPA) with nonresidential investment and changes in business inventories. Output is calculated as the sum of these investment and consumption measures. The quarterly capital-output ratio and the investment-output ratio corresponding to these measures are 6.8 and 0.15, respectively (Veracierto 2008). At stationary equilibrium $I/Y = \delta(K/Y)$, these ratios require the quarterly capital depreciation rate to equal 0.0221. The implied monthly capital depreciation rate is approximately 0.008.

Given the values for β and δ, and given that the capital share in the production function satisfies

$$a = \frac{(1/\beta - 1 + \delta)K}{Y},$$

matching the U.S. capital-output ratio requires choosing a value of a equal to 0.22. Similarly, $b = 0.64$ is selected to generate the share of labor in NIPA.

The aggregate productivity shock is constrained to follow a standard AR(1) process:

$$z_t = \rho z_{t-1} + \varepsilon_t$$

where ε_t is an i.i.d. random variable obeying a normal distribution with mean zero and standard deviation σ. Prescott (1986) selected a value of 0.95 for auto-correlation and a value of 0.00763 for the standard deviation, so the measured Solow residual in the model economy replicates the behavior of the measured Solow residual in the quarterly data. Veracierto (2008) uses the private sector output and capital data and finds a smaller value for the standard deviation, 0.0063. In this paper, we follow the estimates from Veracierto and modify them to suit the period length of one month: ρ is approximately 0.98, and the standard deviation σ is approximately 0.0021.[7]

The parameters that govern the distribution of the capital adjustment costs are β_p, β_q, and the upper bond of capital adjustment costs $\bar{\xi}$. The values of these parameters are chosen to match two pieces of evidence on investment spikes and capital adjustment costs reported by Cooper and Haltiwanger (2006): (1) the proportion of establishments with annual investment rates higher then 20% is about 18.6%; and (2) the average adjustment cost paid relative to the capital stock is 0.0091. To match their observations, we set $\bar{\xi} = 0.028K$, $\beta_p = 1.2$, and $\beta_q = 0.8$.

The marginal disutility of working A is an important determinant of aggregate employment N. Thus, $A = 1.44$ is picked to generate an average employment-population ratio of

[7] $\sigma = 0.0063/\sqrt{3(\rho^4 + \rho^2 + 1)}$

60%, as observed in the data. Since the population is normalized to 1, the average employment level is 0.6. Here, the labor force is assumed to be constant. Since the average unemployment rate in the U.S. data is about 6%, the labor force is 0.64.[8] This means that unemployment is $\tilde{\mathbf{u}}= 0.64 - N$.

The parameter γ is the elasticity of the matching rate with respect to the aggregate recruiting intensity. We use $\gamma = 0.7$, a value close to Shimer's (2005) estimates. Given the value of γ, the technology parameter on the matching function, κ, is then determined by

$$\kappa = h \ / \ (\frac{\tilde{\mathbf{u}}}{\tilde{\mathbf{v}}})^{1-\gamma} \quad ,$$

where $h = M\ (\tilde{\mathbf{v}},\ \tilde{\mathbf{u}})/\tilde{\mathbf{v}}$ is the average job filling rate. The monthly average job filling rate is calculated to be 0.49, consistent with an average vacancy duration of about 45 days.[9]

Since in a stationary equilibrium job creation equals job destruction, $(0.64-\tilde{\mathbf{u}}) * 3.7\% = \tilde{\mathbf{v}} * 0.49$. The monthly average job separation rate is 3.7%, according to data for 2000-2008 from the Job Openings and Labor Turnover Survey (JOLTS, published by the Bureau of Labor Statistics). According to this calculation, the average v-u ratio $\frac{\bar{v}}{\bar{u}}$ is approximately 1.125. [10] Using these values for the v-u ratio and an average job filling rate 0.49, we get a value of 0.508 for κ. The empirical counterpart of vacancy posting cost is difficult to identify. we use a value of 0.15 for e, which implies an average vacancy posting cost of approximately 10% of a month's wage bill.

Finally, the parameter θ, which controls the elasticity of goods consumption, indirectly controls the elasticity of aggregate labor supply. Since, according to the literature, the volatility of aggregate employment is as large as that of aggregate output and the two are

[8] The labor force participation rate for people older than 16 years is about 0.75 in U.S. data. This paper uses 0.64, which is considered as the labor force participation rate for all the people.

[9] $1-(44/45)^{30} = 0.49$. It should be noted that the average duration for vacancies is commonly reported to be under one month, which should imply the job filling rate close to 1. However, as pointed out by van Ours and Ridder (1992), a distinction should be made between the time a help-wanted advertisement is removed and the time it actually takes to fill a vacant position. These authors report that while 75 percent of all vacancies are filled by applicants who arrive in the first two weeks, it takes on average 45 days to select a suitable employee from the pool of applicants. The same target is used in Andolfatto (1996).

[10] The v-u ratio, $\frac{\bar{v}}{\bar{u}}$, is approximately 0.56 in the U.S. data between 2000 and 2008, the level of unemployment, $\bar{u}$, is from the CPS, and the level of vacancy, $\bar{v}$, is from the JOLTS. The monthly average job opening rate is 2.7%. However, according to Davis, Faberman and Haltiwanger (2007), many establishments hire workers during a month in which they report no job openings. They found that at least 36 percent of hires occur without a prior vacancy, as recorded in JOLTS. Since my paper assumes that all establishments post vacancies in order to hire, to have a steady unemployment rate the v-u ratio needs to be higher than that reported in the literature (for instance, Cooper et. al. 2006 use an average v-u ratio of 0.46).

positively correlated, this paper uses $\theta = 0.4$ so that a 1% increase in GDP is associated with a 1% increase in aggregate employment given a positive productivity shock.

Table 5 Key Parameters

Key parameter	Value	Comment
Disutility from working A	1.44	60% employment-population ratio
Capital adjustment cost upper bound $\bar{\xi}$	$0.028K$	18.6% investment spikes
Matching technology κ	0.508	Vacancy duration of 45 days
Matching rate elasticity γ	0.7	Shimer (2005)
Vacancy posting cost e	0.15	10% of one month wage bills
Exogenous job destruction rate φ	3.7%	3.7% job separation rate

7 Results

We lay out a comparative statics analysis to see a simple picture of the model mechanism first, and then proceed to compute the stochastic equilibrium. In the comparative statics analysis we compare two steady states before and after a 1% permanent positive aggregate productivity shock. We picture how job creation and job destruction in small and large establishments respond to an aggregate productivity shock. To compute the stochastic equilibrium, we draw a realization of the calibrated stochastic process for the aggregate productivity. The simulated stochastic model generates the monthly time series of job creation and job destruction in small and large establishments, as well as aggregate GDP. We transfer the monthly time series to the quarterly time series, and then pass the log of these series through an HP-filter with a smoothing parameter 1600 and summarize the cyclical statistics, just like we have done with the U.S. data.

7.1 Comparative Statics

In the comparative statics analysis, we look at the benchmark model with both lumpy capital and labor search frictions, as well as two other experiments: in one experiment, which we call the labor search model, we shut down the lumpy capital margin so the only friction in the model is the labor search friction; in the other experiment, we eliminate the labor market search friction, resulting in a lumpy capital model with Walrasian labor market. The benchmark model successfully predicts the signs of changes in job creation and job destruction in both small and large establishments. Neither the labor search model nor the lumpy capital model can generate the signs that are consistent with the facts.

7.1.1 Benchmark Model

Table 6 shows how job creation and job destruction in small and large establishments respond to a 1% increase in the aggregate productivity in the benchmark model. Job creation in both small and large establishments increases; job destruction in small establishments increases, while job destruction decreases in large ones. The model predicts all the right signs of changes in job creation and job destruction in both small and large establishments.

Table 6 Change of Job Creation and Destruction - Benchmark Model

Job creation		Job destruction	
Small	Large	Small	Large
+1.93%	+0.63%	+3.65%	−1.13%

The reason for the increases in job creation in small and large establishments is obvious. First, the higher aggregate productivity increases the marginal productivity of labor, which leads the establishments to hire more workers. Second, the investment hazard rates increase in response to a positive aggregate productivity shock. This is because the future value of investment increases, which in turn raises the endogenous threshold value for the capital adjustment cost below which establishments undertake investments. As the investment hazard rates increase, more establishments experience growth in capital and hire workers to complement their increased capital. Third, if there is investing, the intensive margin of investment also increases. This also increases the marginal productivity of labor, leading to more hiring.

The different signs of the changes in job destruction in small and large firms can be explained by the coexistence of two opposite effects: the aggregate productivity effect and the composition effect. The aggregate productivity effect reduces job destruction in both small and large establishments, while the composition effect increases job destruction in small establishments but reduces job destruciton in large establishments. The composition effect is caused by different investment responses to aggregate productivity shocks in the presence of the labor market search friction. The labor market search friction creates a wedge between the desired level of employment and the realized level of employment. This wedge is large in small establishments that have invested to increase their size. So a tighter labor market after the increase in productivity does not favor investment in small establishments. On average, after a positive aggregate productivity shock of 1% the investment hazard rate increases by 5% in small establishments and by 14% in large establishments, after accounting for the change in establishment distribution.

The labor market tightness plays a role in shaping the different patterns of job destruction in small versus large establishments. After the permanent positive productivity shock establishments post more vacancies. The labor market becomes tighter (the vacancy-filling rate is low). This tighter labor market makes investment in small establishments less profitable and more risky, since they may very well fail to hire workers quickly to complement the increased lumpy capital. (Note that small establishments, that invest, invest by a larger proportion. The empirical courterpart is that small establishments, that grow, grow faster.) As the tight labor market constrain the future level of employment in the small establishments, their benefit margin of investment is reduced. As a result, a low labor market matching rate holds back some marginal investment projects that would be made by small establishments if the labor matching rate were higher.

As productivity goes up, investment rates increase more strongly in large establishments than in small ones. This causes a decline of the relative marginal labor productivity in small establishments, compared to large establishments. The lower relative labor productivity in small establishments means that labor is more expensive to them. As a result, small establishments destroy more jobs. Although a specific small establishment could create more jobs if it had a good chance to invest, it could also destroy more jobs if it did not invest. The total change in job destruction depends on both the change of investment rates and the intensive margin of job destruction in establishments that do not invest.

The quantitative results show that the change in the intensive margin of job destruction dominates in the group of small establishments. Thus, their job destruction increases after the positive productivity shock. In the group of large establishments the change in investment rates dominates and job destruction decreases. Note that this comparison is conducted between two stationary equilibria, which means that the result represents what happens at the end of an expansion. The result indicates that job destruction in small establishments supports some of the job creation in large establishments when the labor market is tight. This is consistent with the fact reported by Moscarin and Postel-Vinay (2008) that workers flow from small establishments to large establishments during the late phase of an expansion.

7.1.2 Labor Search Model

Here, we shut down the lumpy capital margin to examine a labor search model. The establishments can rent capital without frictions and hire workers from a frictional labor market. In order to have size distribution, we allow for exogeneous productivity differences across establishments. The productivity of an establishment does not change over time, so the distribution of productivity across establishments is time invariant. The productivity distribution is so chosen to generate a similar range of establishment sizes as in the benchmark model. Other parameters are the same as in the benchmark model.

We compare two steady states: the initial steady state and the steady state after a permanent positive productivity shock of 1%. After the shock, job creation and job destruction increase in both small and large establishments. Without lumpy capital, the labor search model cannot generate asymmetric behavior in small versus large establishments as observed in the data. Table 7 shows these responses of job creation and job destruction in small and large establishments. To avoid confusion we should remind the reader that job destruction is the sum of the job losses of all the establishments that have reduced their employment. Job destruction differs from job separation in the current environment , but they are equal in a model where each establishment has only one job. Also note that in the steady state the total number of job creation equals the total number of job destruction.

Table 7 Change in Job Creation and Destruction ——- Labor Search Model

Job creation		Job destruction	
Small	Large	Small	Large
+5.33%	+48.05%	+4.97%	+48.7%

With labor search frictions, after the shock, establishments post more vacancies. This means that unemployment rate is lower, the labor market is tighter, and the vacancy-filling rate is lower, as shown in table 8. Without capital market frictions, all the differences in job creation and job destruction across establishments come from the differences in the labor search and matching outcomes. Since all the establishments post more vacancies, some of them succeed in the labor market, creating more jobs. Since the failure rate increases because of a tigher labor market, more establishments fail to hire and have to suffer the loss of workers (assumed exogeneous separation). So the random labor search and matching causes the increases in both job creation and job destruction: job creation increases in the group of establishments that are lucky to hire workers, while job destruction increases in the group of establishments that fail to hire.

Table 8 Aggregate Conditions in the Labor Search Model

	Before shock	After shock
Lowest wage (A/p)	1.4152	1.4272
Unemployment rate	6.00%	5.48%
Vacancies / labor force	5.95%	6.33%
Vacancy-filling rate	0.3733	0.3082

7.1.3 Lumpy Capital Model

The lumpy capital model is built with a frictional capital market but a Walrasian labor market. Most of the parameters in this model are the same as in the benchmark model, but the marginal disutility of working A and the capital adjustment costs are adjusted so the unemployment rate is 6% and the size distribution of establishments resembles that in the benchmark model. Table 9 shows the changes in job creation and destruction in small and large establishments after a 1% permanent productivity shock. The model generates counterfactual predictions: job destruction increases in large establishments, but job creation in large establishments does not increase.

Table 9 Change in Job Creation and Destruction - Lumpy Capital Model

Job creation		Job destruction	
Small	Large	Small	Large
+1.31%	+0.00%	+0.00%	+2.44%

In the lumpy capital model, the changes in investment hazard rates in small and large establishments after a productivity shock lead to changes in job creation and job destruction. The reason is simple. Job creation comes from establishments that invest and hire to complement the increase in capital, and job destruction comes from establishments that do not invest and fire workers because of capital depreciation, or that lose workers because of exogenous job-worker separation. After a 1% permanent positive productivity shock, the investment hazard rates in the group of small establishments increase more than those in the group of large establishments. On average, the investment hazard rates increase by 5.6% more in the group of small establishments in the lumpy capital model without labor search frictions. So the group of small establishments can increase its job creation by a higher rate. If this increase in job creation in small establishments is very strong, it is possible that the small establishments will steal workers from the large establishments. Thus, job creation in large establishments does not increase, but job destruction does.

The aggregate amount of job destruction in each group of establishments depends on both the proportion of establishments that experience job destruction and the magnitude of that job destruction. Although the proportion of establishments that experience job destruction decreases in the group of large establishments, the magnitude of job destruction increases because of increased wages. The overall effect in the experiment is that job destruction increases in the group of large establishments. The increased job destruction is a result of relatively lower marginal labor productivity and relatively higher cost of workers. This abnormal response of large establishments is, however, consistent with the literature on the augmented RBC models with real rigidities. This literature finds that the labor input tends to decline in response to a positive technology shock via strong general equilibrium effects (see Francis and Ramey 2004, and Hashmat and Tsoukalas 2006).

The main difference made by adding the labor market search frictions into the lumpy capital model is that it changes the investment hazard rates in small relative to large establishments. The investment hazard rates of small establishments are relatively lower when the labor market is frictional. In other words, during booms, the labor market search frictions deter some investment projects in small establishments, while favoring the larger establishments. Moreover, when the representative-establishment model is altered only by adding random capital adjustment costs, the resulting stationary establishment size distribution is quite unrealistic; there are too few small establishments, and too many large establishments. So adding random labor search increases the mass of small establishments because some establishments fail to hire and stay small.

7.2 Stochastic Equilibrium and Cyclical Statistics

The cyclical statistics generated by the benchmark model are shown in table 10. In this experiment, the magnitude of numbers is secondary compared to their sign. This is because the data is measured at the firm level, but the model works at the establishment level, and it assumes that the larger establishments belong to the larger firms. This is a strong assumption, although the empirical evidence shows a strong firm-establishment size correlation.

Table 10 The Cyclical Statistics of Job Creation and Destruction

	Job creation		Job destruction	
	Small	Large	Small	Large
	Correlation with GDP			
Data	0.6192	0.4498	0.1815	−0.1781
Model	0.5324	0.3332	0.4222	−0.1230
	Relative standard deviation [11]			
Data	2.51	6.18	2.51	6.81
Model	6.20	4.64	5.08	4.82

In general, the cyclicality generated by the benchmark model is consistent with the data. The model performs best at matching the positive correlation between job destruction in small establishments and GDP. The model cannot generate the relatively smaller volatility found in small establishments. This is not surprising since, in the real world, small establishments face more risks, so they enter and exit more frequently even in good times[12] This is not captured by the model. The more idiosyncratic the risks faced by small establishments (during both booms and busts), the smaller the volatility of their job creation and job destruction caused by the aggregate shock.

[11] The standard deviations of the variables are divided by the standard deviation of GDP.

[12] Davis, Haltiwanger and Schuh (1996) show that the exit rates of small firms are still high during expansion.

Besides changing investment hazard rates in the model, the random labor market search frictions have other effects on job creation and job destruction. First, they make the larger establishments destroy more jobs during recessions. The individual establishment job-filling rates are different even if all the establishments post the same number of vacancies. So random matching makes the establishment size distribution more dispersed. Specifically, random matching makes establishments with an identical capital stock have different levels of employment. Among the establishments with the same capital stock, the larger ones are always more affected by a negative aggregate productivity shock. This contributes to the countercyclical job destruction in large establishments.

8 Concluding Remarks

The paper incorporates random labor market search frictions into a lumpy capital model in which capital adjustment is subject to idiosyncratic costs. In this model, the history of investment and labor market search outcome fully determines the sizes of firms. The same factor market frictions and uncertainties that generate the firm size distribution affect small and large firms differently. In such an economy, the aggregate productivity shocks are propagated through the frictional factor markets and, therefore, affect the employment dynamics in small and large firms asymmetrically.

By combining labor market search frictions and capital adjustment frictions, this paper finds that both the investment decision (whether to invest or not) and the intensive margin of investment depend on firm size. Moreover, the investment hazard rate in large firms responds strongly to positive aggregate productivity shocks. The labor market search frictions deter marginal investment projects in small firms, especially when the labor market is tight. So the investment hazard rate in small firms does not increase as much as in large firms during booms. This generates a worker movement from small firms to large firms during booms, and contributes to the surprising procyclical job destruction in small firms.

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