Neoclassical Models of Endogenous Schumpeterian Growth:

A Model of Growth through Creative Destruction

Author: Aghion & Howitt

Zhe Li

SUFE

Fall 2010

Process of Creative Destruction

"The fundamental impulse that sets and keeps the capitalist engine in motion comes from the new consumers' goods, the new methods of production or transportation, the new markets,….

[This process] incessantly revolutionizes the economic structure *from within*,

incessantly destroying the old one,

incessantly creating a new one.

This process of Creative Destruction is the essential fact about capitalism."

Joseph Schumpeter (1942): *Schumpeterian Growth*

The “Creative Destruction”

- **On the positive side**,
  - Can individual firms’ innovation cause sustained growth?
- **On the normative side, if indeed individual firms’ innovation is an important engine of growth**
  - Externality of one firm’s research on others?
  - Relationship between current research and future research?
  - Whether government intervention improves welfare or increases growth rate?

Aside: a research paper

- Observe facts
- Facts are important
- Model

Growth through Creative Destruction

- Vertical innovations (quality ladders): Industrial innovations which improve the quality of products
  - Romer (1990): horizontal product innovations, using the Dixit-Stiglitz (1977) model of product variety, involve no obsolescence
- Creative destruction: better products render previous ones obsolete
  - Modeling the innovation process as in the patent-race literature Tirole (1988, Ch. 10):
    - Successful firms acquire a patent that will grant them monopoly over the innovation
    - New successful firms destroy the old ones
- Uncertain innovations: research firms succeed randomly

Environment

- Two goods:
  - Consumption goods (final goods)
  - Intermediate goods: input in producing final goods
- Three sectors:
  - Final goods production
  - Intermediate goods production
  - Research sector: increases the quality of intermediate goods (therefore the productivity of final goods production)

Environment

- A continuum of infinitely-lived individuals with identical preference

$$u(y) = \int_0^\infty e^{-r\tau} y_\tau d\tau$$

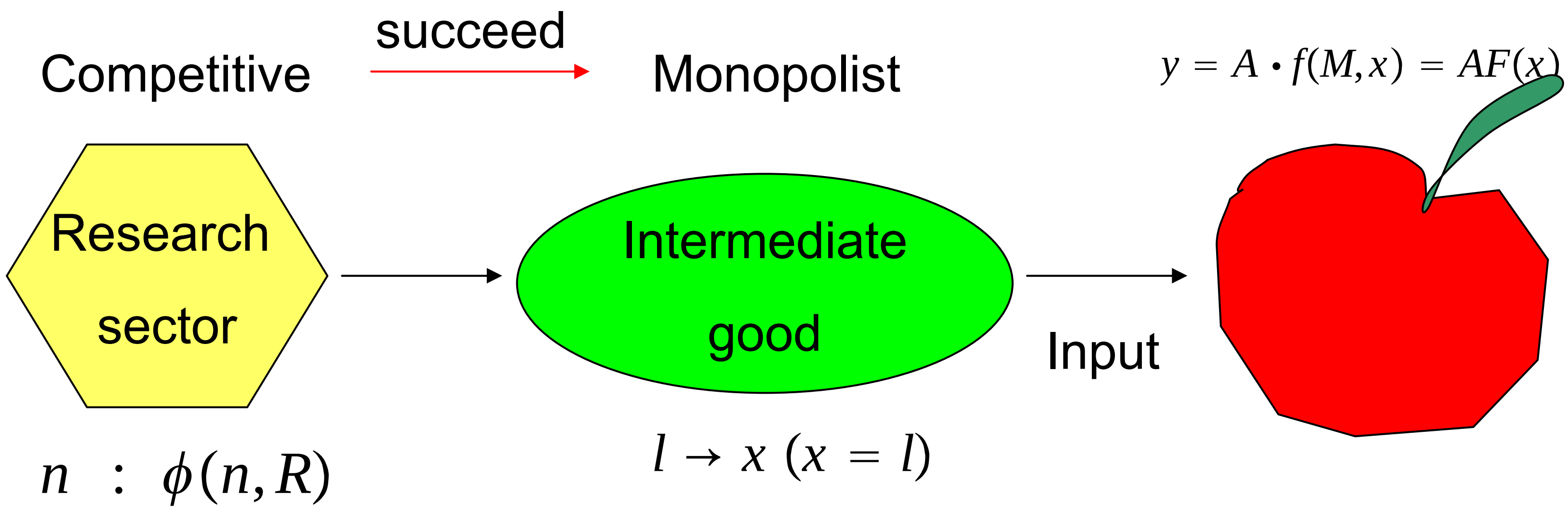
- Constant rate of time preference r>0, r is the only parameter determines intertemporal allocation of consumption goods
- Risk-neutral: remove the motive to use capital markets for risk-sharing, leave the main intertemporal relationship to creative destruction
- Without physical capital

Environment cont'd

- No disutility from supplying labor
  - Unskilled labor, which can be used only in producing the consumption good (mass M)
  - Skilled worker, which can be used either in research or in the intermediate sector (mass N)
  - Specialized worker, which can be used only in research

    (mass R)
- Each worker is endowed with a one-unit flow of labor

The Three Economic Sectors



Incentive for Innovation: The innovator of one period becomes the exclusive user of the new technology in the intermediate sector in the next period.

Problem for Economy: Allocation of the given skilled labor pool (N) between R&D (n) and intermediate good production (*l*). $N = n + l$

Innovation and Output

The final-good sector produces goods using an intermediate input, t denotes the generation of the intermediate good, productivity A_t is associated with x_t

$$y_t = A_t F(x_t)$$

$$F' > 0, \; F'' < 0$$

An innovation brings a new generation of intermediate goods. Use of the new intermediate good increases the productivity parameter A by a constant factor (A_0 is the initial value) :

$$A_t = A_0 \gamma^t \qquad \gamma > 1$$

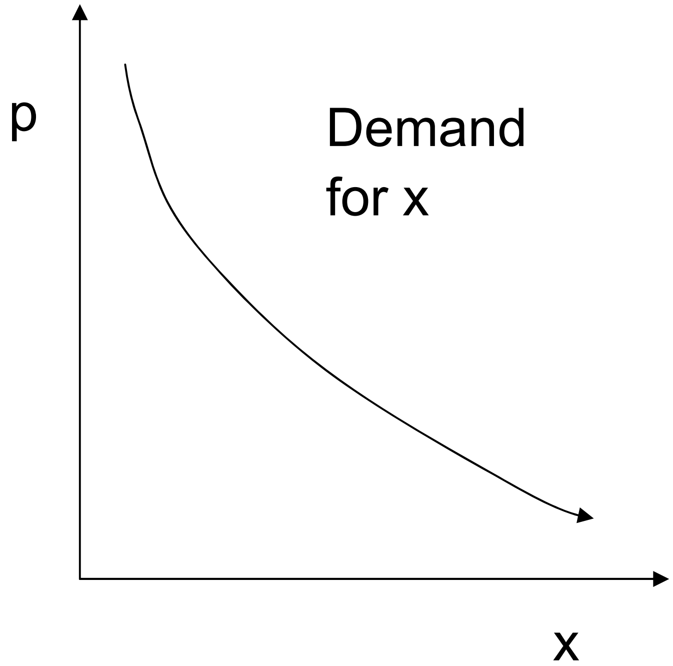
Final Goods Sector

The final-good producer (consumer good is numeraire)

$$\max_{x_t} A_t F(x_t) - p_t x_t$$

$$p_t = A_t F'(x_t)$$

i.e. final-good producers will employ x until its marginal product equals its price



Intermediate Monopolist's Decision

Monopolist chooses the level of output, *x*, that maximizes his profits:

$\max_x [px - wl]$ $\quad x = l$

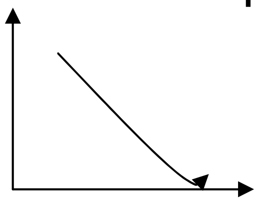
$\max_x [AF'(x)x - wx]$ $\quad$ taken the demand function as given

$\max_x [A(F'(x)x - \omega x)]$ $\quad$ denote $\omega = \frac{w}{A}$ as the productivity adjusted wage

Assume $\frac{d^2(xF'(x))}{dx^2} < 0$, i.e. $2F''(x) + xF^3(x) < 0$

f.o.c.: $\omega = \frac{d(xF'(x))}{dx} = F'(x) + xF''(x) \equiv W(x)$

$x = W^{-1}(\omega) = X(\omega)$ $\quad$ Given that $W'(x) < 0,\ X'(\omega) < 0$



Supply of the intermediate good (demand for skilled labor) decreases as real wage increases

Intermediate Monopolist's Profit

Profit decreases as real wage increases

$$A\pi(\omega) = A[X(\omega)F'(X(\omega)) - \omega X(\omega)]$$

$$\pi(\omega) = [X(\omega)F'(X(\omega)) - \omega X(\omega)]$$ recall $\omega = F'(x) + xF''(x)$

$$= -[X(\omega)]^2 F''(X(\omega))$$

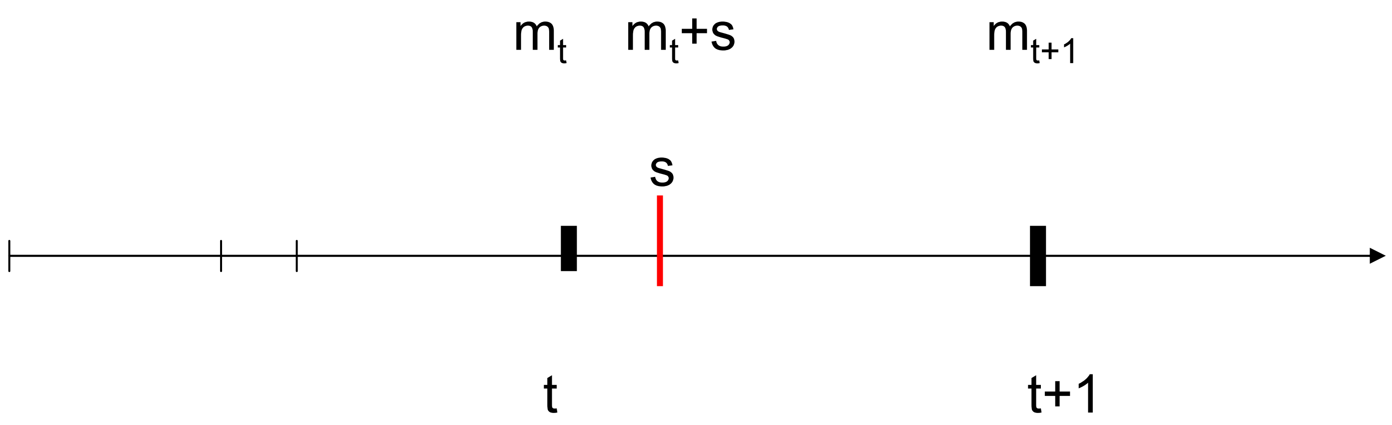
$$\pi'(\omega) = -X(\omega)X'(\omega)[2F''(x) + xF^3(x)] < 0$$

given $X'(\omega) < 0$ and $2F''(x) + xF^3(x) < 0$

Wage and General Equilibrium Effect

- We have derived that $\pi'(\omega) < 0$
- Wage is determined by the demand of skilled labor in both research and intermediate sector.
- Consider the future (after next innovation): higher demand for future research labor will push future wage ω_{t+1} up, thereby decreasing the flow of profits π_{t+1}
- The prospective innovator's profit is the motivation for current research
- A lower expected future profit will tend to discourage current research, that is, to drive the worker hired in research n_t down.
- higher n_{t+1} ➔ lower n_t through wage

t^{th} innovation and time s



Generation (innovation) t

At time s after the t^{th} innovation, the value of a monopolist (t^{th} innovator) is $V_t(s)$

The Research Sector

A firm employing the amounts n, R of the two factors in research will experience innovations with a Poisson arrival rate of $\lambda\phi(n, R)$

ϕ has constant returns

total flow of specialized labor must equal R in equilibrium

$\varphi(n) = \phi(n, R)$

$$\max_{n_t} \lambda\varphi(n_t)V_{t+1}(0) - w_t n_t - w_t^R R$$

Kuhn-Tucker conditions:

$w_t \geq \lambda\varphi'(n_t)V_{t+1}(0), \; n_t \geq 0$

with at least one equality

$V_{t+1}(0)$ is the value of the $t+1^{st}$ innovation

Discrete the Continuous Value

Look at a very small time interval Δ

$$V_t(s) = \frac{1}{1+r\Delta}[A_t\pi_t(s)\Delta + \lambda\varphi(n_t(s))\Delta \cdot 0 + (1-\lambda\varphi(n_t(s))\Delta)V_t(s+\Delta)]$$

The value of holding an asset (license on the t[th] innovation at time s) is, equal to the discounted value of the profit flow that lasts for a time period Δ, plus the expected value of being replaced (the flow probability of this loss is the arrival rate $\lambda\varphi(n_t(s))\Delta$, plus the continuation value $V_t(s+\Delta)$ (after a time period Δ the current innovator will continue to hold the license) with probability $(1-\lambda\varphi(n_t(s))\Delta)$

Discrete the Continuous Value

$$V_t(s) = \frac{1}{1 + r\Delta}[A_t\pi_t(s)\Delta + \lambda\varphi(n_t(s))\Delta \cdot 0 + (1 - \lambda\varphi(n_t(s))\Delta)V_t(s + \Delta)]$$

$$(1 + r\Delta)V_t(s) = A_t\pi_t(s)\Delta + (1 - \lambda\varphi(n_t(s))\Delta)V_t(s + \Delta)$$

$$r\Delta V_t(s) = A_t\pi_t(s)\Delta - \lambda\varphi(n_t(s))\Delta V_t(s + \Delta) + V_t(s + \Delta) - V_t(s)$$

$$rV_t(s) = A_t\pi_t(s) - \lambda\varphi(n_t(s))V_t(s + \Delta) + \frac{V_t(s + \Delta) - V_t(s)}{\Delta}$$

$$As\ \Delta \to 0,\ rV_t(s) = A_t\pi_t(s) - \lambda\varphi(n_t(s))V_t(s) + \frac{dV_t(s)}{ds}$$

$if\ \frac{dV_t(s)}{ds} = 0,$ which is true since Poission distribution of new innovation independent of time

$$V_t(s) = \frac{A_t\pi_t(s)}{r + \lambda\varphi(n_t(s))}$$

The Effect of Creative Destruction

$$V_{t+1}(s) = \frac{A_{t+1}\pi_{t+1}(s)}{r+\lambda\varphi(n_{t+1}(s))}$$

The denominator is the *obsolescence-adjusted interest* rate and shows the effect of creative destruction.

The more research is expected to occur, the shorter the likely duration of the monopoly profits, and hence the smaller the payoff to innovating.

The Monopolist Does not Do Research

- *Arrow/replacement effect:*
  - The reason why the monopolist chooses to do no research is that
    - the value to the monopolist of making the next innovation would be $V_{t+1} - V_t$, (destroy itself)
    - which is strictly less than the value V_{t+1} to an outside firm.

Compensation Scheme Discourages Research

- *Intertemporal spillover effect (current research has positive effect on future research and growth):*
  - An innovation raises productivity forever. It allows each subsequent innovation to raise A_t by the same multiple γ.
  - The producer of an innovation captures (some of) the rents from that productivity gain, but only during one interval.
  - After that the rents are captured by other innovators, building upon the basis of the present innovation, but without compensating the present innovator

Future Research and Current Research

$$\text{if } n_t > 0,\ A_t W(x_t) = \lambda \varphi'(n_t) V_{t+1}(0)$$

$$recall \quad W(x_t) = \omega = \frac{w_t}{A_t},\ W'(x_t) < 0$$

$$W(N - n_t) = \lambda \varphi'(n_t) V_{t+1}(0) / A_t$$

$$V_{t+1}(0) = \frac{A_{t+1} \pi_{t+1}(0)}{r + \lambda \varphi(n_{t+1}(0))}$$

$$\frac{W(N - n_t)}{\lambda \varphi'(n_t)} = \frac{\gamma \pi \left(W(N - n_{t+1}) \right)}{r + \lambda \varphi(n_{t+1})}$$

Equilibrium Research Labor

$$\frac{W(N-n_t)}{\lambda\varphi'(n_t)} = \frac{\gamma\,\pi\left(W(N-n_{t+1})\right)}{r+\lambda\varphi(n_{t+1})}$$

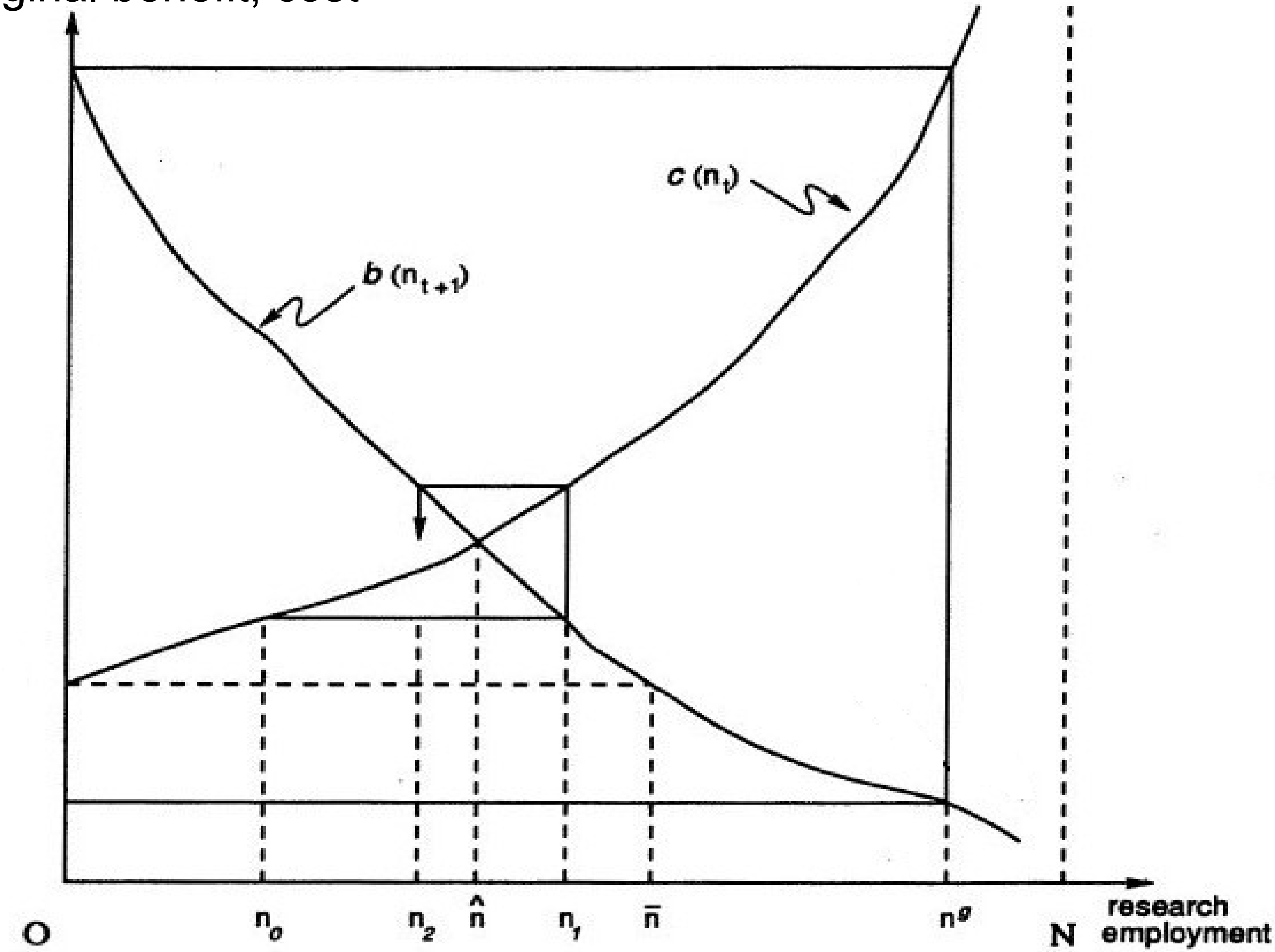
Cost of innovation normalized by how fast it comes: $c(n_t)$

Discounted profit stream of an innovator: $b(n_{t+1})$

$$c'(n) > 0 \qquad b'(n) < 0$$

$$n_t = \psi(n_{t+1}) \qquad \psi' < 0$$

Marginal benefit, cost



A Stationary Equilibrium

$$\hat{n} = \psi(\hat{n})$$

Balanced Growth:

$$y_t = A_t F(N - \hat{n})$$

$$y_{t+1} = \gamma y_t$$

Balanced Growth

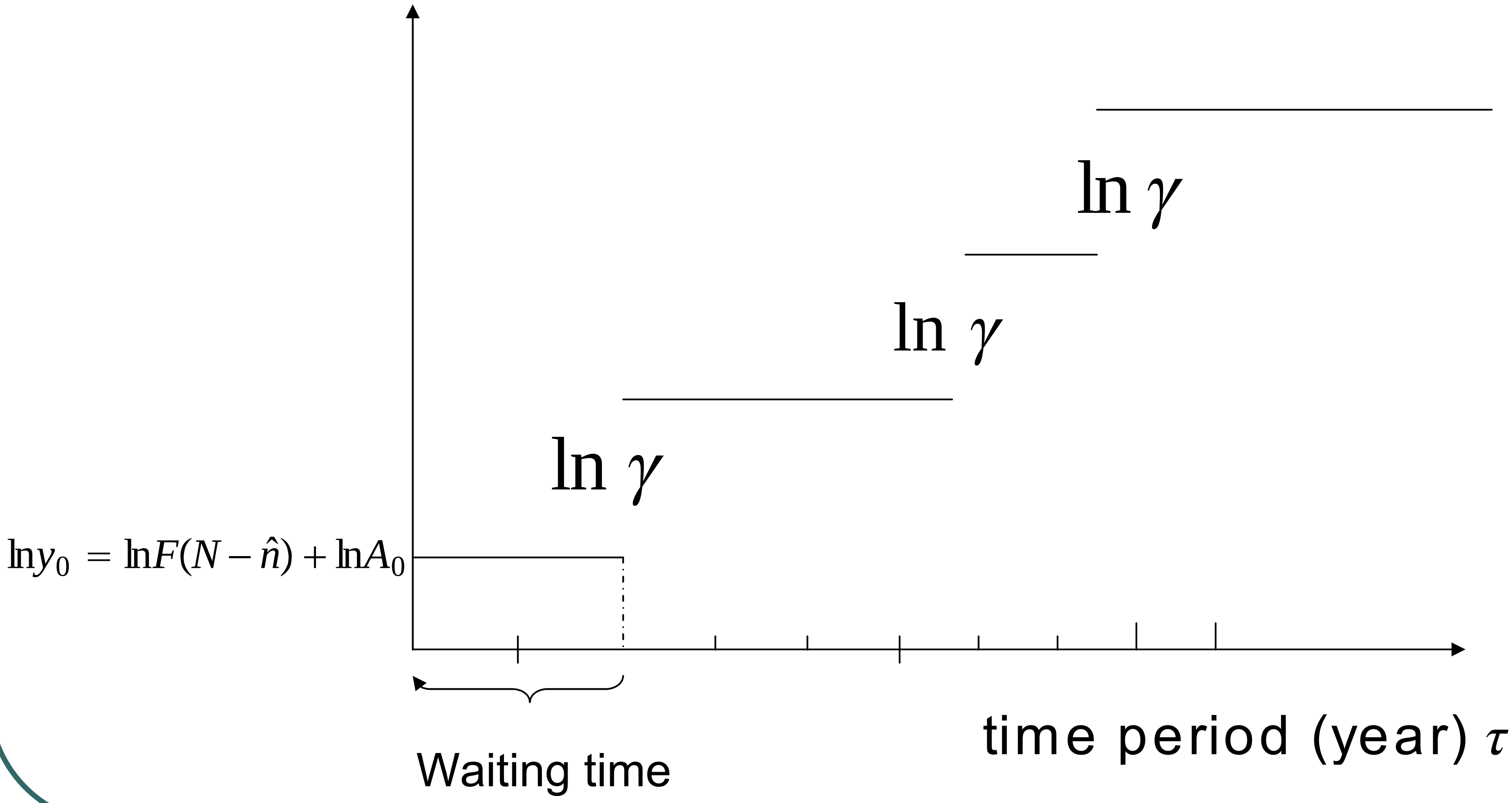
$$y_t = A_t F(N - \hat{n})$$

$$y_{t+1} = \gamma y_t \qquad \lambda \varphi(\hat{n})$$

The time path of the log of real output

$$\ln y_0 = \ln F(N - \hat{n}) + \ln A_0$$

A Random Step Function



Observations at Discrete Points in Time 1 Year Apart

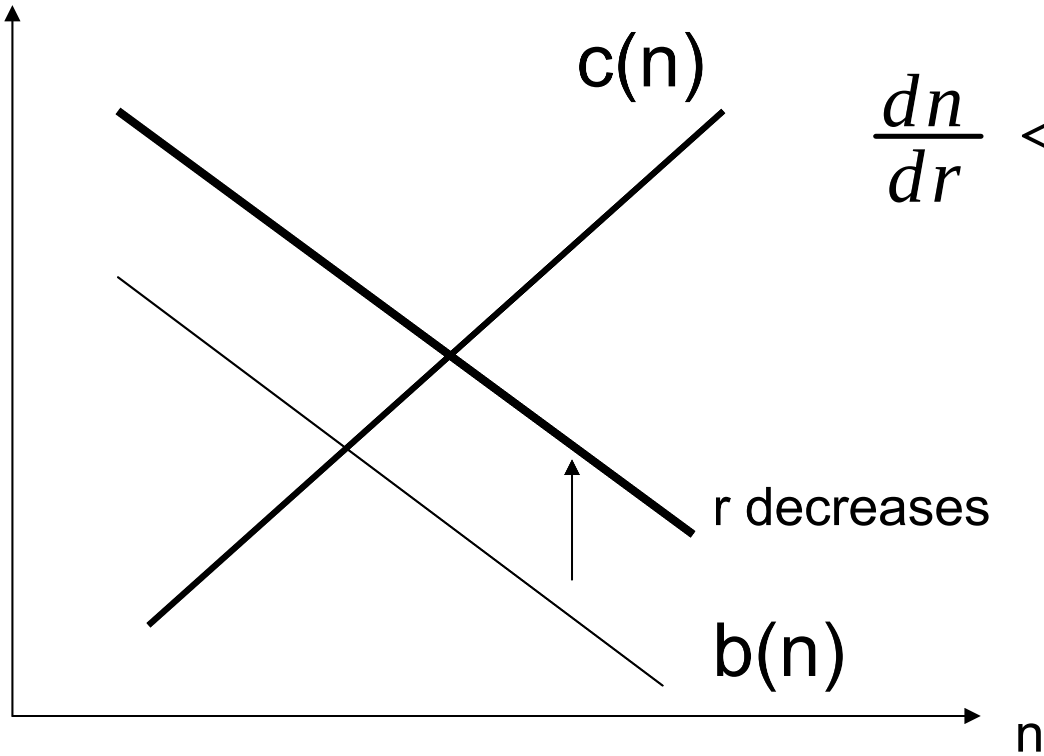
$$\ln y(\tau + 1) = \ln y(\tau) + \varepsilon(\tau)$$

$$\varepsilon(\tau) = \ln \gamma \times \#\text{ of innovation between } \tau \text{ and } \tau + 1$$

$$E[\varepsilon(\tau)] = \lambda \varphi(\hat{n}) \ln \gamma$$

$$Var[\varepsilon(\tau)] = \lambda \varphi(\hat{n})(\ln \gamma)^2$$

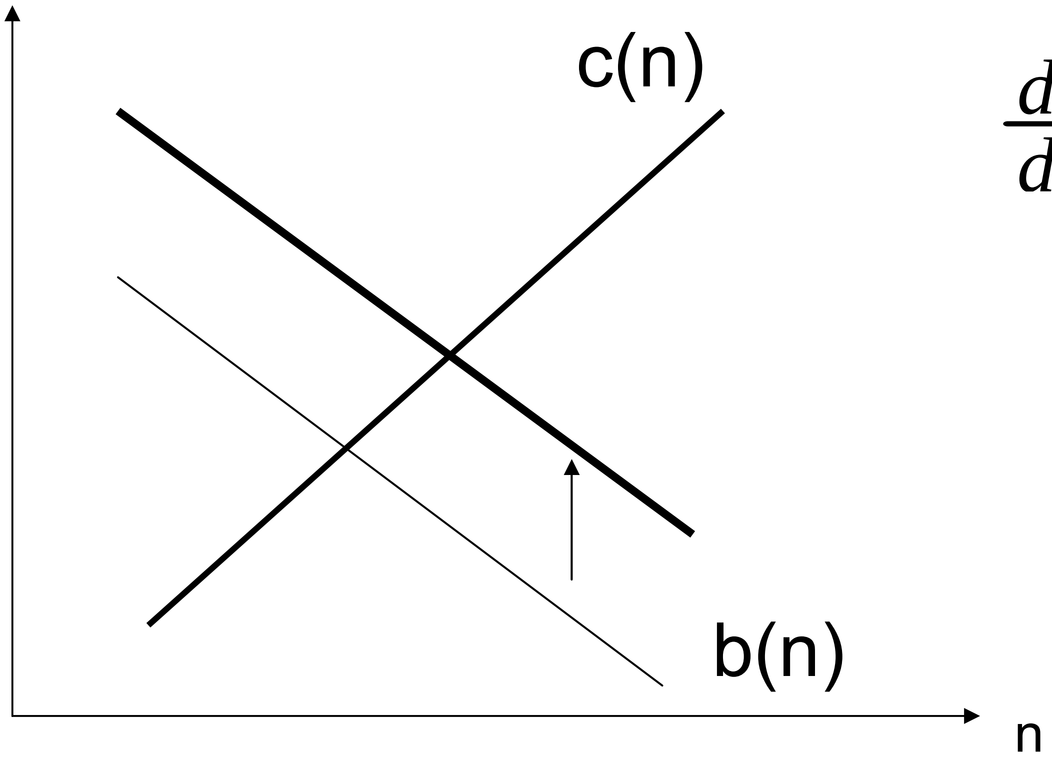
Comparative Statics
Research in stationary equilibrium



$$\frac{dn}{dr} < 0, \quad \frac{dn}{d\gamma} > 0$$

$$\frac{W(N-n)}{\lambda\varphi'(n)} = \frac{\gamma\ \pi\ \left(W(N-n)\right)}{r + \lambda\varphi(n)}$$

Comparative Statics
Research in stationary equilibrium

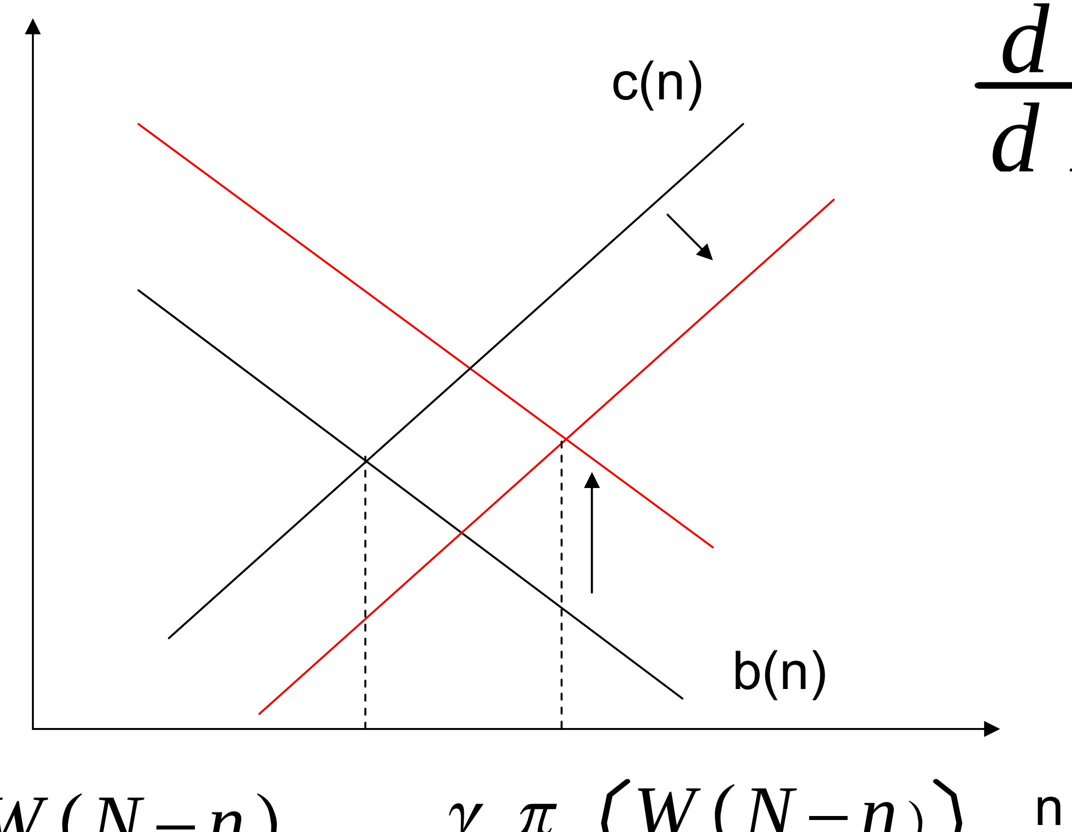


$$\frac{dn}{d\lambda} > 0$$

$$\frac{W(N-n)}{\varphi'(n)} = \frac{\gamma\,\pi\left(W(N-n)\right)}{\frac{r}{\lambda} + \varphi(n)}$$

Comparative Statics
Research in stationary equilibrium

$$\frac{dn}{dN} > 0$$



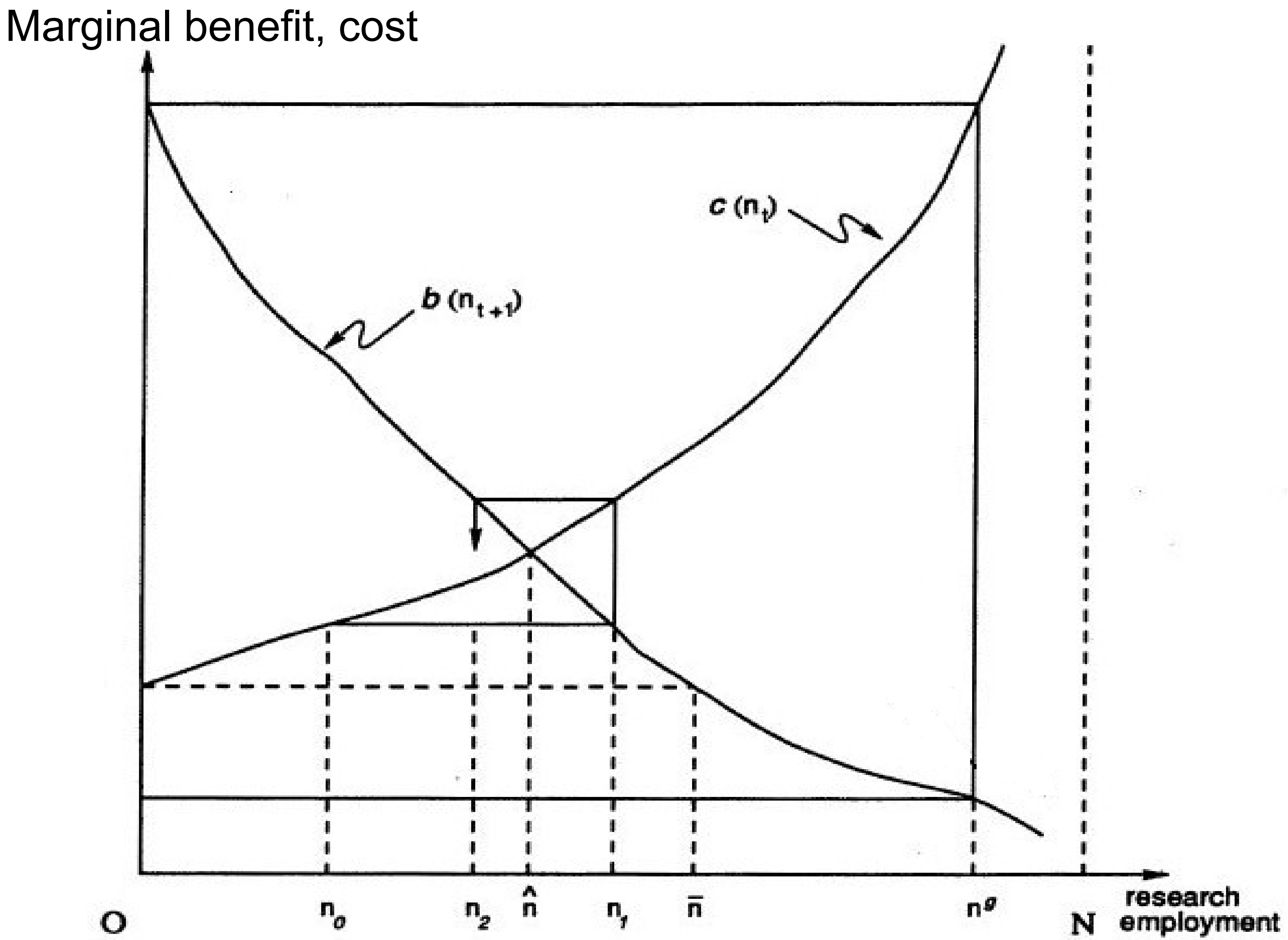
$$\frac{W(N-n)}{\lambda \varphi'(n)} = \frac{\gamma \pi \left(W(N-n)\right)}{r + \lambda \varphi(n)}$$

A Perfect Foresight Equilibrium

a sequence $\{n_t\}_0^\infty$ satisfying

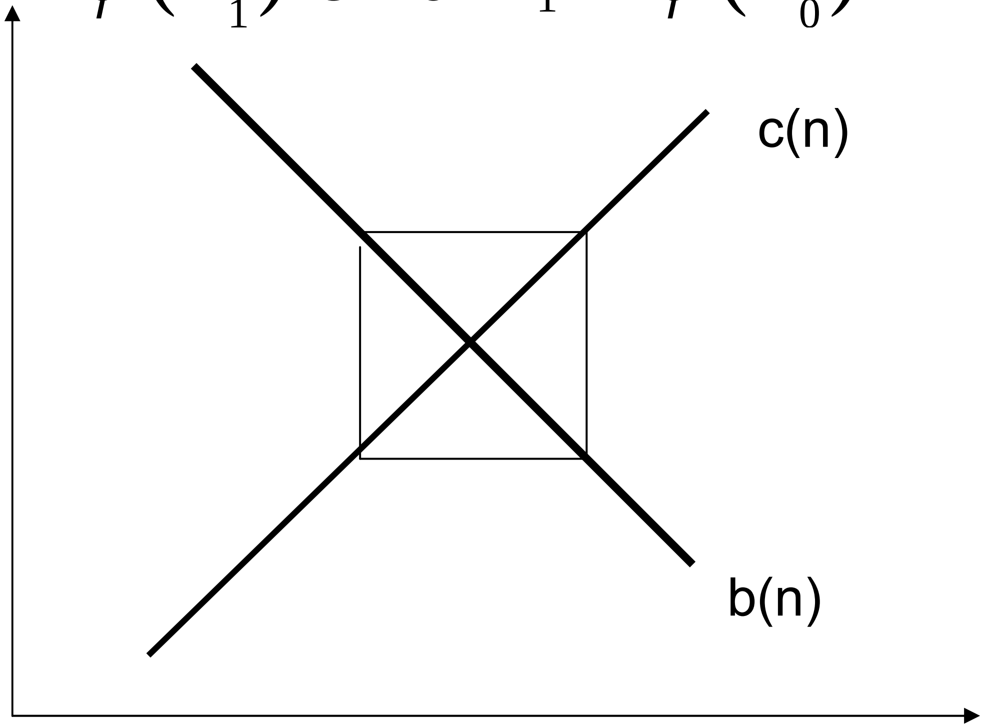
$$n_t = \psi(n_{t+1})$$

$$c(n_t) = b(n_{t+1})$$


Marginal benefit, cost
c (n_t)
b (n_t+1)
O
n_0
n_2
n̂
n_1
n̄
n^g
N
research employment

A Two-Cycle Equilibrium

A two-cycle is a pair (n_0, n_1) such that $n_0 = \psi(n_1)$ and $n_1 = \psi(n_0)$.



Social Planner's Problem

Let $A_t u_t(s)$ be the discounted value of utility from $m_t + s$ onward:

$$rA_t u_t(s) = y_t + \lambda\varphi(n_t)[A_{t+1}u_{t+1}(0) - A_t u_t(s)] + \frac{d}{ds}[A_t u_t(s)]$$

$$ru_t(s) = F_t + \lambda\varphi(n_t)[\gamma u_{t+1}(0) - u_t(s)]$$

$$u_t(s) = \frac{F_t + \gamma\lambda\varphi(n_t)u_{t+1}(0)}{r + \lambda\varphi(n_t)}$$

Steady State

$$u_t(0) = u_t(s) = \frac{F_t + \gamma\lambda\varphi(n_t)u_{t+1}(0)}{r + \lambda\varphi(n_t)}$$

at steady state $u_t(0) = u_{t+1}(0) = u$

$$u = \frac{F(N-n)}{r-(\gamma-1)\lambda\varphi(n)}$$

Social Optimal Research

$$\max_n u = \max_n \frac{F(N-n)}{r-(\gamma-1)\lambda\varphi(n)}$$

f.o.c. $[r-(\gamma-1)\lambda\varphi(n)]\cdot F' + F\cdot(\gamma-1)\lambda\varphi'(n) = 0$

$$\frac{F'(N-n^*)}{\lambda\varphi'(n^*)} = \frac{(\gamma-1)F(N-n^*)}{r+\lambda\,\varphi(n^*)-\gamma\lambda\,\varphi(n^*)}$$

Compare Social Optimal and Market Equilibrium --- Four Differences

$$\frac{F'(N-n^*)}{\lambda\varphi'(n^*)} = \frac{(\gamma-1)F(N-n^*)}{r+\lambda\varphi(n^*)-\gamma\lambda\varphi(n^*)}$$

$$\frac{W(N-n)}{\lambda\varphi'(n)} = \frac{\gamma\pi\left(W(N-n)\right)}{r+\lambda\varphi(n)}$$

(1) Discount rate:

The social rate is less than the rate of interest, whereas the private rate is greater

➔ $n^* > n$

Compare Social Optimal and Market Equilibrium --- Four Differences

$$\frac{F'(N-n^*)}{\lambda\varphi'(n^*)} = \frac{(\gamma-1)F(N-n^*)}{r+\lambda\varphi(n^*)-\gamma\lambda\varphi(n^*)}$$

$$\frac{W(N-n)}{\lambda\varphi'(n)} = \frac{\gamma\pi\left(W(N-n)\right)}{r+\lambda\varphi(n)}$$

(2) Output versus Profit :

➔ $n^* > n$

Compare Social Optimal and Market Equilibrium --- Four Differences

$$\frac{F'(N-n^*)}{\lambda\varphi'(n^*)} = \frac{(\gamma-1)F(N-n^*)}{r+\lambda\varphi(n^*)-\gamma\lambda\varphi(n^*)}$$

$$\frac{W(N-n)}{\lambda\varphi'(n)} = \frac{\gamma\pi\left(W(N-n)\right)}{r+\lambda\varphi(n)}$$

(3) Business Stealing: $\gamma - 1$ instead of γ

➔ $n^* < n$

Compare Social Optimal and Market Equilibrium --- Four Differences

$$\frac{F'(N-n^*)}{\lambda\varphi'(n^*)} = \frac{(\gamma-1)F(N-n^*)}{r+\lambda\varphi(n^*)-\gamma\lambda\varphi(n^*)}$$

$$\frac{W(N-n)}{\lambda\varphi'(n)} = \frac{\gamma\pi(W(N-n))}{r+\lambda\varphi(n)}$$

(4) "Monopoly – distortion" effect:

Social cost of research employment exceeds the private cost

$$W = F' + xF''$$

➔ $n^* < n$

Endogenous Size of Innovation

Aiming for innovation of size γ

Arrival rate: $\lambda\varphi(n) \cdot \upsilon(\gamma)$

$$\upsilon'(\gamma) < 0,\ \upsilon''(\gamma) < 0$$

The bigger the innovation,

the harder it is to discover

Stationary Payoff to innovator

$$V_{t+1} = \frac{A_{t+1}\pi(\hat{\omega})}{r+\lambda\varphi(\hat{n})\upsilon(\hat{\gamma})}$$

$\hat{\gamma}$ is the stationary-equilibrium value of γ.

$\lambda\varphi(\hat{n})\upsilon(\hat{\gamma})$ The probability of being replaced by other innovators

A Research Firm's Expected Return

$$V_{t+1} = \frac{A_{t+1}\pi(\hat{\omega})}{r + \lambda\varphi(\hat{n})\upsilon(\hat{\gamma})}$$

A research firm takes $\lambda\varphi(\hat{n})\upsilon(\hat{\gamma})$, the activity of other firms, as given

It chooses a γ, (may $\neq \hat{\gamma}$, out of the equilibrium)

$$A_{t+1} = \gamma A_t,\ V_{t+1} = \gamma V_t$$

Market Solution

$$\max_{\gamma} \lambda \varphi(n) \upsilon(\gamma) \gamma V_t - w_t n_t - w_t^R R$$

$$\frac{d}{d\gamma}[\upsilon(\gamma)\gamma] = 0$$

$$\upsilon(\hat{\gamma}) + \hat{\gamma}\upsilon'(\hat{\gamma}) = 0$$

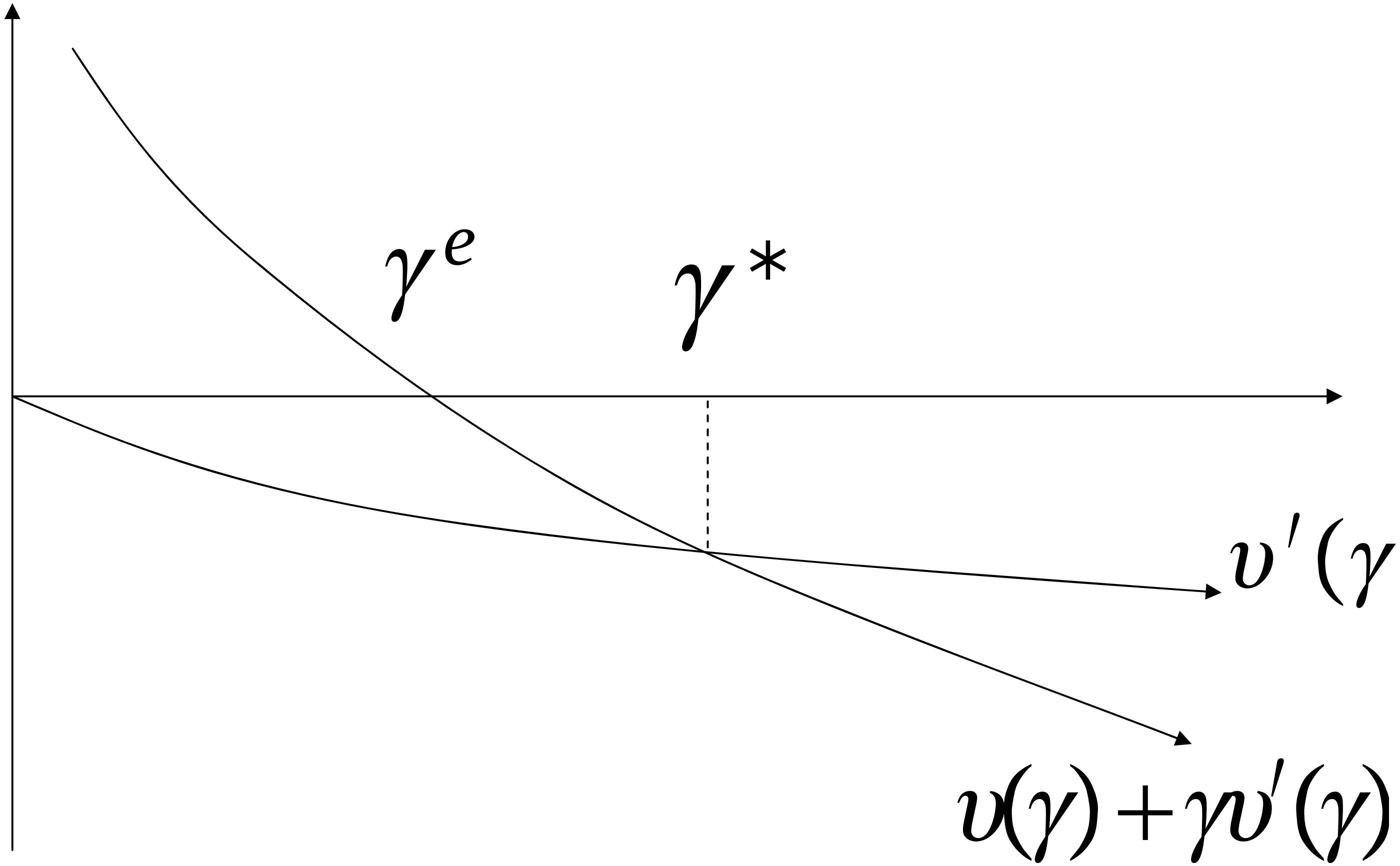
Social Planner

$$U = \frac{A_0 F(N-n)}{r - \lambda \varphi(n) \upsilon(\gamma)\gamma}$$

$$\max_{\gamma} U$$

$$\upsilon(\gamma^*) + \gamma^* \upsilon'(\gamma^*) - \upsilon'(\gamma^*) = 0$$

Social and Market Size of Innovation



Aside: what if $\frac{dV_t(s)}{ds} \neq 0$

$$\frac{dV_t(s)}{ds} = [r + \lambda\phi_t(n_t(k))]V_t(s) - A_t\pi_t(s)$$

define $z_t(s) = V_t(s)e^{-\alpha(s)}$ as the discounted present value

$\alpha(s) = \int_0^s [r + \lambda\phi_t(n_t(k))]dk \qquad \dot{\alpha}(s) = r + \lambda\phi_t(n_t(s))$ adjusted interest rate

$$\frac{dz_t(s)}{ds} = [\dot{V}_t(s) - \dot{\alpha}(s)V_t(s)]e^{-\alpha(s)} = -A_t\pi_t(s)e^{-\alpha(s)}$$

$z_t(s) = -\int_s^\infty dz_t(k) = \int_s^\infty A_t\pi_t(k)e^{-\alpha(k)}dk,$ if $z_t(\infty) = 0$

$$V_t(s) = \int_s^\infty A_t\pi_t(k)e^{-[\alpha(k)-\alpha(s)]}dk$$