License Complementarity and Package Bidding: The U.S. Spectrum Auctions*

Mo Xiao[†] Zhe Yuan[‡]

Nov. 19, 2018

Abstract

The U.S. spectrum licenses cover geographically distinct areas and are often complementary to each other. A bidder seeking to acquire multiple licenses is then exposed to risks of winning only isolated patches. To allocate licenses more efficiently, the Federal Communications Commission allowed bidders to bid for (predefined) packages of licenses in Auction 73. We estimate the magnitude of license complementarity by modeling the bidding process as an entry game with interdependent markets and evolving bidder belief. Bidders' decisions on bidding (and not bidding) provide bounds on licenses' stand-alone values and complementarity between licenses. We estimate the total complementarity to be around two thirds of the total bidding ($19 billion) in Auction 73. Complementarity in a 1 MHz nationwide license is worth $918 million to an average large bidder but only $120 million to an average small bidder. Our counterfactual analysis shows that the effects of package bidding on bidders' exposure risks depend on package format and package size. More importantly, package bidding increases FCC revenue substantially at the cost of reducing bidder surplus and increasing license allocation concentration.

Keywords: Spectrum Auctions, Complementarity, Package Bidding, Moment Inequalities

---

*We thank seminar participants at Toulouse School of Economics, the University of Massachusetts Amherst, 2018 International Industrial Organization Conference, 2018 Shanghai University of Finance and Economics IO summer school, 2018 Montreal IO conference, and 2018 Colombian Workshop on Structural IO for comments and suggestions. We have especially benefited from discussion with Brent Hickman, Ariel Pakes, and Chenyu Yang. We are grateful for financial support from the NET institute summer grant.

[†] Eller College of Management, University of Arizona. Email: mxiao@eller.arizona.edu.

[‡] Alibaba Group. Email: yyyuanzhe@gmail.com.

1 Introduction

In the United States, firms who intend to provide cell phone services bid for spectrum licenses in simultaneous multi-round ascending auctions hosted by the Federal Communication Commission (FCC). These licenses are often complementary to each other: a bidder may have higher willingness to pay if it is able to obtain two or more licenses together. To facilitate more efficient license allocation, FCC has started in the mid 2000s to allow bidders to make a single bid for a group of licenses — a "package." For example, in auction 73, conducted in early 2008, FCC divided the country into different packages in Block C and designated the entire country as a single package in Block D.[1]

One key reason why FCC started package bidding is to alleviate the "exposure problem" that often arise in previous spectrum auctions, which allows only *a la carte* license bidding. Take for example a bidder with the highest willingness to pay for 20 MHz of spectrum covering the entire United States. As auction rounds increase and bidding prices increase, this bidder faces a tough choice. If the bidder continues to bid for all licenses, it is "exposed" to the risk that it will acquire only a few of the licenses. Without complete nationwide coverage, these few licenses will not be worth the high prices the bidder has bid. If the bidder decides to withdraw from bidding, it may fail to acquire licenses for which it actually has the highest value. In either case, the license allocation is inefficient.[2]

At first glance, package bidding does seem to be able to alleviate this exposure problem. If FCC allows package bidding, it is all or nothing so a bidder will no longer face the uncertainty of winning only a few licenses within a package. The exposure problem generated by a bidder seeking to win multiple *packages*, however, remains and could become exacerbated. More importantly, whether package bidding improves social surplus depends on the distribution of stand-alone values across bidders. For example, suppose two bidders are bidding for two licenses, A and B, and the value of complementarity between the two licenses is \$5 million for both bidders. Suppose the stand-alone values of bidder 1 on license A and B are \$10 million and \$2 million respectively, while the stand-alone values of bidder 2 on license A and B are \$2 million and \$10 million, just the opposite of bidder 1's values. If FCC allows only *a la carte* bidding, then bidder 1 will win license A, bidder 2 will win license B, and the social surplus (equal to bidder surplus plus FCC revenue in this case) will be \$20 million. If FCC auctions off license A and B in a single package, then either bidder can win this package and

---

[1] Auction 73 auctions off the rights to operate the 700 MHz radio frequency band in the United States in 5 blocks: A, B, C, D, and E, each block covering different frequencies.

[2] Bulow, Levin, and Milgrom (2009) provides a detailed description of the exposure problem faced by a new entrant and points out that the source of this problem is the uncertainty about the final auction prices.

the social surplus is \$17 million, \$3 million lower than under *a la carte* bidding.

Package bidding is still in its explorative stage. Evidence that package bidding improves efficiency comes mostly from laboratory experiments – whether these conclusions can be extrapolated to real world bidding is subject to continuing controversy. To design for package bidding, FCC needs information on two important empirical objects: one is the magnitude of complementarity across licenses and the other is the distribution of stand-alone values across bidders. Estimating complementarity across licenses is a challenging task: each license is no longer an independent observation. As bidders have almost infinite combinations of licenses to bid for under constantly evolving belief about winning any set of license, it is infeasible to calculate bidders' optimal strategy on which licenses to bid for and how much to bid.[3]

This paper takes on this challenge. We use FCC Auction 73, the first auction in which FCC designates packages to be auctioned off.[4] This auction has 1,099 license for sale in total of 5 blocks, covering 698-806 MHz band (therefore called "700 MHz Band") and averaging 6-22 MHz per license. Between January 24 and March 18, 2008, 214 bidders participated the auction, of which 101 successfully won at least one license. These bidders are highly heterogeneous, including telecommunications giants such as AT&T and Verizon, regional carriers such as Cellular South, as well as small, obscure small firms that qualifies for FCC's steep bidding credit. In total, Auction 73 raised \$18.958 billion for FCC, selling 1,090 licenses. This auction has a "limited disclosure" format, meaning that bidders do not observe competing bidders' identities in the bidding process. This format simplifies bidder belief formation process and strategy space because it reduces the dimensions of information bidders can access to form and update belief. We analyze data from the middle of the auctions, when early noises have dissipated and bidders bid as if one's current bids in the auction are going to set the final prices. In FCC's language, this is "straightforward bidding" — FCC has devised many rules (details in section 2) to eliminate strategic bidding and to facilitate "straightforward bidding."

As the vast majority of bidders bid exactly the FCC designated minimum acceptable price in each round of the auction, we model bidding decisions on these interdependent licenses as an entry game with interdependent markets. In this entry game, we estimate bidders' belief about final winning probabilities in every round of bidding. As bidding prices rises, bidders' provisional winning sets change, and bidders' beliefs based on bidding history evolve, bidders'

---

[3] For example, Auction 73 has 1,099 licenses for sale. A bidder seeking a national footprint has $2^{1,099}$ combinations of potential strategies. In comparison, Cantillon and Pesendorfer (2006) has no more than 3 London bus routes in an auction, and Kim, Olivares, and Weintraub (2014) has about 30 units in each auction of Chilean school meals.

[4] Auction 51 is the first FCC auction that allows bidders to assemble packages themselves.

decision on bidding (and not bidding) gives us bounds on licenses' stand-alone values and complementarity.

Specifically, it is the bidders' change of bidding decisions that are most informative. When a bidder starts to bid on a license in the middle of the auction, the stand-alone value of this license has not changed but the price for this license has become higher. The only reason the bidder starts bidding must be that the bidder now has a higher expected value of this license's contribution to the set of licenses the bidder perceives to win. This revealed preference characterizes the lower bound of the complementarity magnitude. Similarly, when a bidder stops bidding on a license, we obtain the upper bound of the complementarity magnitude. With complementarity identified from bidders' change of bidding behaviors on the same license, bidders' decisions on different licenses at a certain round gives us bounds on the stand-alone values, which are determined by the difference between prices and expected complementarity. In short, we construct moment inequalities in fashion of Pakes, Porter, Ho, and Ishii (2015), using bidders' revealed preferences over different rounds on the same license to estimate complementarity and those across different licenses in the same round to estimate standard-alone values.

Our estimation of complementarity is based on a total of 148,445 (bidder-license-round) inequalities, and our estimates satisfy 66% of these inequalities. We estimate complementarity value to be around \$2.4 billion per block. Aggregating over 5 blocks, the complementarity amounts to two thirds of the total bidding (approximately \$19 billion) in this auction. Large bidders value complementarity more than medium and small bidders. Complementarity in a 1 MHz nationwide license is worth \$918 million to an average large bidder, \$222 million to an average medium bidder, and only \$120 million to an average small bidder. We find similar bidder heterogeneity in bidders' stand-alone values, although the difference is not statistically significant. Heterogeneity at the license level, including bandwidth, population, population density, and cellphone tower density, plays a much larger role in bidders' stand-alone values.

With estimates on stand-alone values and complementarity, we perform counterfactual experiments to assess welfare trade-offs if FCC designates different packages in block B, in which licenses have the smallest geographic coverage. We perform two alternative aggregations of these small geographic areas. With these packages, FCC can either exercise pure bundling (henceforth "pure package"), in which only package bidding is allowed, or mixed bundling (henceforth "mixed package"), in which both package bidding and *a la carte* bidding are allowed. For each simulation, we evaluate welfare tradeoffs by calculating FCC revenue selling off licenses, bidders' surplus acquiring these licenses, the magnitude of the exposure problem, and final license allocation to different types of bidders.

We have three major findings. First, package bidding only alleviates the exposure problem to a limited extent, and this effect depends on package format and package size.[5] This is because although package bidding eliminates within-package exposure problem, it aggravates between-package exposure problem. A bidder bidding on large packages faces the risks of not winning all the packages the bidder aims for. Decomposing a bidder's *ex-post* regret, we can see that the largest chunk of the exposure problem without package bidding comes from the *ex-post* regret from large bidders' premature withdrawal. With package bidding (especially with mixed package bidding), the "should not have bid" regret increases but the "should have bid" regret decreases more, leading to smaller *ex-post* regret overall. The effect is especially prominent for large bidders, meaning that large bidders bid more aggressively under package bidding and especially under mixed package bidding.

Second, package bidding increases total social welfare at the cost of redistributing bidders' gain to FCC revenue. This redistribution effect is more prominent in mixed package than in pure package and when FCC bundles licenses into large packages. The driving force of this result is large bidders' more aggressive bidding behavior under mixed package bidding, which pushes up final bidding prices and allows FCC to gain substantially in revenue. Among the three types of bidders, the large bidders have the most to lose under package bidding. In total, the package bidding regimes increase total social surplus, and mixed package fares slightly better than pure package. These results confirm Cantillon and Pesendorfer (2006): the welfare consequences of allowing package bidding are ambiguous, depending on the distributions of valuations and the size of the complementarity effect.

Lastly and very importantly, although bidders are generally worse off in package bidding, package bidding allocates many more licenses to large bidders at the expense of medium and small bidders. Overall, package bidding creates a more concentrated market structure (especially when packages cover large geographic areas). The implications of these results, as they concern future competitive landscape in the cellphone market, are not included in our previous welfare assessment. One of FCC's goals is to create and maintain a more even play field for future cellphone providers (this goal is embodied by the steep bidding credit FCC grants to small bidders) — package bidding actually works against this goal, which contradicts our previous result that only assesses short-term welfare tradeoffs. Overall, due to these ambivalent welfare effects, we caution policy makers about the adoption of package bidding.

---

[5] If FCC assembles licenses into covering very large geographic areas and allow mixed package at the same time, the exposure problem even becomes larger.

Structure estimate of valuation functions of FCC spectrum auctions is sparse, particularly because estimating complementarity across licenses is a challenging task.[6] Of this literature we are closest to our predecessors Fox and Bajari (2013) and Yeo (2009), which are the only two papers allowing license complementarity in structural estimation of FCC auctions.[7] Fox and Bajari (2013) estimates a matching game in which the equilibrium outcome of matches is pairwise stable (as in Fox (2010)), using bidding data in the final rounds of 1995-1996 C block auction. The C Block divided the continental U.S. into 480 small, geographically distinct licenses, and Fox and Bajari's research finds that four large regional licenses would raise the allocative efficiency substantially. When they incorporate price information in their estimation, they find the value of complementarity from a nationwide license is \$120 billion, which they claim as "absurdly high" (totally bids for the C block amount to only \$10 billion) and caution readers about their result. Yeo (2009) also uses moment inequalities to estimate bidder valuations in Auction 66. Her goal is to evaluate markups of the winning bidders and to assess the level of competition in FCC spectrum auctions, which is very different from ours. In Yeo (2009)'s model bidders believe that they will win all the licenses they bid in each round, and she obtains very wide, inconclusive interval estimates for license complementarity.

Most of the literature on FCC spectrum auctions investigates bidders' strategic behaviors.[8] Cramton and Schwartz (2002) find in an early FCC spectrum auction, a small fraction of bidders tag the last three digits of their bids with the market number of a related license to signals to other bidders on which licenses to bid or not bid. This collusive bidding behavior results in significant FCC revenue loss. Doraszelski, Seim, Sinkinson, and Wang (2017) look into the case that FCC conducts a reverse auction for broadcast TV licenses: FCC acquires spectrum from broadcast TV license holders and then repacks the acquired spectrum to mobile broadband spectrum. Their paper finds that multi-license holders of broadcast TV licenses strategically withhold some licenses to increase the price for their remaining licenses.[9] Our work has a different focus. As FCC tightened rules since 2000s to regulate spectrum auctions and alleviate collusive concerns (more details in Section 2),

---

[6] Hong and Shum (2003) initializes the line of literature. They model bidding for each license as a single-unit auction and do not consider complementarity across licenses.

[7] Ausubel, Cramton, McAfee, and McMillan (1997) and Moreton and Spiller (1998) find strong reduced-form evidence on geographic license complementarity in U.S. spectrum auctions.

[8] There is also an extensive literature on collusion in auctions, to name a few recent ones, Asker (2010), Conley and Decarolis (2016), Kawai and Nakabayashi (2015).

[9] Strategic bidding is also a common theme in the literature on multi-unit auctions, which involve the sale of many related items such as treasury, spectrum, electricity, and emission permits. Notable empirical examples along this lines include Wolfram (1997), Hortaçsu and Puller (2008), Hortaçsu, Kastl, and Zhang (2018) among many others. Hortaçsu (2011) discusses empirical analysis of multi-unit auctions.

bidders bid more straightforwardly and we can recover bidders' valuation of licenses and the complementarity among licenses by characterizing bidder behaviors with simple rules. Our goal is use these model primitives to evaluate the effects of potential policies that could improve the allocative efficiency of spectrum auctions.

Our empirical strategy is inspired by Haile and Tamer (2003), which characterizes an incomplete model and use only necessary conditions for identification and estimation. As Haile and Tamer (2003) points out, English auctions lack sufficient structure to yield a tractable theoretical model without significant abstractions. They construct bounds based on two simple assumptions: bidders bid no more than their valuations nor let another bidder win at a price lower than their valuations. We follow this guiding principle: We do not invert the first-order conditions derived from a specific model to recover valuations from observed bids; instead, we use necessary conditions of an incomplete model to simplify the complex process of price bidding. We add to Haile and Tamer (2003) contributions in two fronts: 1) We take their insight to multiple-units auctions where there is complementarity among these units; 2) We allow "revealed preferences" approach to reveal both bidder preferences and bidder belief. Our approach allows us to deal with both the computational and statistical curse of dimensionality that arises when the number of items increase in a combinatorial auction. Our work complements our predecessors (Cantillon and Pesendorfer (2006), Kim, Olivares, and Weintraub (2014)) that studies first-price combinatorial auctions, all of which exploit bidders' first-order conditions in their profit maximization problems. The most recent paper in this small but growing literature is Gentry, Komarova, and Schiraldi (2018), which establishes non-parametric identification of primitives in simultaneous first-price auctions with bidders' preferences over combinations and estimates cost synergies in Michigan highway procurement auctions.

The paper is organized as follows. In Section 2 and 3, we present stylized facts in FCC spectrum auctions and especially in Auction 73 to explain how they affect our modeling choices. In Section 4, we outline a model, formulate behavioral assumptions characterizing the solutions to the model, and describe how we construct moment inequalities from these behavioral assumptions. In Section 5 and 6, we present estimation results, conduct counterfactuals, and discuss counterfactual results and implications.

2 FCC Spectrum Auctions

2.1 Spectrum Auctions Basics

Since 1994, FCC has conducted auctions of licenses to transmit signals over specific bands of the electromagnetic spectrum. These auctions are open to any eligible company or individual that submits an application and upfront payment, and is approved to be a qualified bidder by FCC. In the thirty years since, FCC has conducted 89 auctions and raised hundreds of billion dollars from spectrum auctions. With technology advancement that enables the use of different frequency, FCC has introduced new bandwidth to auctions four times, as the telephone industry transitions from landlines to mobile devices and then to 3G and 4G data networks. Top auctions include Auction 35 (C and F Block Broadband Personal Communications Service), which garnered \$16 billion in FCC revenue, Auction 66 (Advanced Wireless Services (AWS-1)) garnering \$14 billion, Auction 73 (*apropos* of this study) \$19 billion, and Auction 97 (Advanced Wireless Services (AWS-3)) \$41 billion.[10] Major participants include incumbent cellphone carriers such as AT&T and Verizon, potential entrants into wireless business such as the Dish Network, technology companies such as Google,[11] and even individual citizens. Most spectrum auctions, especially recent ones, are conducted electronically and results are accessible over Internet-based bidding system.[12] In a large auction, hundreds of firms bid for thousands of licenses in hundreds of rounds over a few weeks of time.

FCC spectrum auction adopts the format of simultaneous multiple-round ascending auction. "Simultaneous" means that all licenses are up for bidding at the same time throughout the entire auction. "Multiple-round" refers to discrete, successive rounds, the length of each round specified in advance by FCC. "Ascending" means that in each round, FCC adds a small increment to the standing winning bid (typically 10-20% of the standing winning bid) to determine the acceptable minimum bid of a new round. There is no predetermined number of rounds. The auction will continue until a round occurs in which all bidder activities cease — no new bids, withdrawals of bids or use of proactive activity waivers (explained later in this section). This round becomes the closing round of the auction and determines the final outcome of winners and winning bids.

FCC strictly controls information released before and after each round of bidding. Depending on information released after each round of bidding, there are two types of auctions:

---

[10] Personal Communication Services refers to the bandwidth from 1850 MHz to 1990 MHz, and Advanced Wireless Services refers to the range between 1710 MHz and 2180 MHz.

[11] For example, Google participated in Auction 73 in an effort to encourage open access of spectrum.

[12] Telephone bidding is also available for some qualified bidders.

"full disclosure" and "limited disclosure" auctions. In "full disclosure" auctions, after each round closes, FCC releases information on bids placed by all bidders. In "limited disclosure" auctions, information release by FCC is very restricted. In Auction 73, a "limited information" auction for example, FCC withholds information until after the close of the auction that concerns the identifies of potential bidders, including, among other things, license selections, upfront payments and eligibility information (FCC (2008)). After each round closes, FCC only notifies bidders the number of bids placed for a license in the round and the provisionally winning bids for this license. Bidders are forbidden to share or discuss bidding strategies at any time. In this case, bidding is strictly anonymous. Before the adoption of "limited information" format, FCC's flexible auction format and released information on bidding can be exploited by bidders so they bid more strategically or even collude with each other. This is why FCC started to exercise tight control of information since mid 2000s.

Moreover, FCC has auction rules to help reducing strategic bidding and bidding collusion. The most notable rules are eligibility requirement and activity requirement(Bomberger (2007)). FCC assigns each license a fixed number — "bidding units" — to measure bidder eligibility and activity. Before an auction starts, each bidder must submit an upfront payment that determines its bidding eligibility in the auction. A bidder's eligibility is specified by the maximum number of bidding units the bidder's upfront payment allows it to be active on in a given round.[13] Bidder eligibility cannot be increased after the bidder submits its upfront payment. After bidding starts, FCC measures a bidder's activity by the sum of bidding units of the licenses on which the bidder places bids in the current round and licenses on which the bidder has provisionally winning bids from a previous round. Activity requirement dictates that a bidder must bid on a specified portion of its maximum eligibility at a given round (80% and later 95% in Auction 73). If the activity requirement is not met, FCC reduces the violating bidder's eligibility permanently, possibly preventing the bidder from further bidding in the auction.[14] In a real-time or continuous auction, bidders sometimes wait until the last minute to place their bids for different reasons. For example, on eBay bidders often "snipe" the auctioned item at the last seconds leading to the concluding time of the auction. FCC's eligibility and activity requirement prevent super slow bidding and sniping bidder, and more generally, bidders' strategic use of postponing bidding decision. In short, FCC has

---

[13] The upfront payment does not limit the dollar amount a bidder may bid for any license.

[14] A bidder has a limited number of "proactive" waivers to use to satisfy the activity requirement. In Auction 73, this number is three. The use of waivers preserves a bidder's current eligibility. FCC will automatically apply a waiver for a bidder (instead of a bidder applies waivers proactively, which keeps the auction open) at the end of a round in which the bidder's activity is too low, unless the bidder has no waivers left or the bidder reduces eligibility voluntarily.

implemented rules to ensure that participants bid actively throughout the auction.

2.2 License Complementarity and Package Bidding

Each license is associated with a geographically distinctive area, "Market Area" in FCC terminology. FCC has five types of Market Areas. Going from small to large in geographic coverage, they are: Cellular Market Areas (CMAs), Basic Trading Areas (BTAs), Basic Economic Areas (BEAs), Major Trading Areas (MTAs), and Regional Economic Area Groupings (REAs). A typical CMA covers only 3 to 4 counties, a BEA roughly 15 counties, while a REA can cover hundreds of counties and span several states. In different splits, United States are divided into 734 CMAs or 176 BEAs or 12 REAs. FCC carefully balances geographic coverage associated with a license and its allotted bandwidth to avoid market power concentration and to induce entry into high-cost, sparsely-populated rural areas.[15]

Spectrum licenses, especially the ones covering adjacent geographic areas, are complementary to each other. Complementarity exists when the value of the whole is greater than the sum of the parts. It is clearly the case in spectrum auctions because a typical cellphone carrier provides continuous coverage over at least a certain region. What is not clear, from both FCC's perspective and industry experts', is the magnitude of such complementarity over a combination of licenses. A major justification of the *a la carte* auction format is to allow bidders to bid for a group of licenses they would like to acquire together. For example, if a regional firm aims at the entire California market, it may want to bid for all California licenses at a certain bandwidth together in each round of bidding; if a national firm aims to fix bandwidth congestion in its current service areas, it may want to bid for more bandwidth for areas where the congestion problem is more severe. The magnitude of license complementarity, an economic primitive that determines the allocative efficiency of FCC spectrum auctions, is usually a bidder's private information and so seems best to be left at the free disposal of bidders themselves.

Problems arise, however, with the free assembly a bidder needs to decide on during the auction process. A typical bidder seeks to acquire multiple licenses, which have values higher than the sum of all stand-alone values due to complementarity. As bidders are never certain about the set of licenses they will win until auction concludes, they are subject to the risks of either "overbidding" or "underbidding" *ex-post* regret. "Overbidding" regret happens when a bidder becomes the winner of isolated patches of licenses, for which it has values lower

---

[15] Generally, there is a negative relationship between the size of the frequency bandwidth and that of the geographic coverage: licenses of narrower spectrum bandwidth is usually assigned a larger market area, and vice versa.

than its winning bids. In contrast, "underbidding" regret happens when a bidder gives up bidding on a license for which it has higher value than a rival bidder's winning bid. Both types of *ex-post* regret constitute the "exposure problem," which is the result of license complementarity and bidder uncertainty about auction allocation outcome.

One way to fix the problem is to allow bidders to submit bids on packages of licenses. FCC first proposed a simple form of package bidding in 2000 (Auction 31), but the implementation did not happen until 2003 (Auction 51).[16] Following Auction 51, FCC revised the package bidding format and implemented it in Block C, Auction 73, the auction set of this study. The most significant change is that the packages are predefined by FCC, instead of being proposed by bidders.[17] For the 12 licenses in the C Block, FCC permits *a la carte* bidding as well as package bidding.

FCC and industry experts believe package bidding, in general, should be an improvement upon individual license bidding, especially when there is strong complementarity among licenses. As the revised FCC package bidding format endows FCC the power of setting packages for bidding, understanding the magnitudes of license complementarity becomes a key policy input. The goal of this paper is to provide estimates of license complementarity to help FCC make better decisions on how to divide licenses and set packages for bidding.

3 Auction Set 73

Auction 73 began on January 24, 2008 and closed on March 18, 2008. The spectrum frequency to be auctioned off ("the 700 MHz band") had been occupied by television broadcasters and may then be used for flexible fixed, mobile, and broadcast uses. A total of 1,099 licenses were offered in 5 blocks: 176 Basic Economic Area (BEA) licenses in the A and E Blocks, 734 Cellular Market Area (CMA) licenses in the B Block, 12 Regional Economic Area Grouping (REA) licenses (and 3 packages) in the C Block, and one nationwide license in the D Block. After 261 rounds (in a matter of 38 days), 206 out of 214 qualified bidders placed at least one bid and 101 of them won 1,090 licenses.[18] Gross bids amounted to \$19,120,378,000, from which FCC actually received \$18,957,582,150 (net bids, which is the amount FCC receives after credits and discounts to bidders).

This is a "limited disclosure" auction, meaning that bidder identities information is with-

---

[16] Auction 51 is a small auction in which one bidder won five licenses (in one package) in three rounds.

[17] Bidder-defined package bidding becomes an intractable problem as the scale of bidding, the number of licenses and potential packages as well as the number of bidders, increases.

[18] FCC held the remaining 9 licenses and auctioned some off in later auctions.

held by FCC until the auction concludes. In any round and for any license, bidders know the characteristics of the license, the minimum acceptable bid by the FCC, the number of bids placed in all previous rounds, and the provisionally winning price bid. Bidders know the provisionally winning licenses they have at each round but they receive no information who has what licenses before the auction concludes.

In the following subsections, we report and analyze descriptive statistics of Auction 73. The statistics characterize an auction with highly heterogeneous licenses and equally, if not more so, heterogeneous bidders. More importantly, we highlight stylized facts which affect our model choices in subsequent sections.

3.1 Heterogeneous Licenses

Auction 73 consists of 5 blocks of licenses. Table 1 reports summary statistics for different blocks of licenses and top 10 licenses. As shown in the table, these licenses vary substantially in bandwidth and geographic area coverage, two most important characteristics of spectrum licenses. A license with wider bandwidth and larger geographic area (usually corresponding to larger population) is more valuable.[19] In these 5 blocks, the bandwidth goes from 6 MHz (Block E) to 22 MHz (Block C), and the geographic coverage goes from CMA (roughly 3 to 4 counties) to NWA (Nationwide Area, the first time FCC offers a license covering the entire United States). Block A licenses have the same geographic delineation as Block E licenses, but the doubled bandwidth in Block A draws more bidders and much higher bids than Block E. Although having much smaller geographic coverage than Block D, Block C licenses seems to be of more value due to its 22 MHz of bandwidth compared to Block D's mere 6 MHz.

Some blocks draw much more competition than others. Block B draws the most number of bidders while Block D only has one.[20] Block C is the most interesting block to this study, as FCC auctions off predefined packages in this block. Bidders can bid for 12 REAs licenses separately or they can bid for 3 packages separately: package 50 states (REA 1-8), package Atlantic (REA 9 and REA 11) and package Pacific (REA 10 and REA 12). Google bids \$1.037 billion in round 1 for package 50 states and increase its bids over time. In round 17, Google bids \$4.713 billion for package 50 states. From round 27 to round 30, Verizon enters, places 8 different bids on REA 1 to REA 8, and wins 7, spending \$4.742 billion in total. Google gave up afterwards and Verizon eventually becomes the biggest winner of Block C,

---

[19] All else equal, licenses covering densely-populated areas typically have higher value than sparsely-populated ones.

[20] Qualcomm is the only bidder in Block D. Qualcomm's provisionally winning bid (\$472 million) for the D Block license, however, does not meet FCC's reserve price (\$1.3 billion)

Table 1: Licenses, Bids and Winning Bids

					Bid			Win			
Block	# of Licenses	Market Area	Bandwidth (MHz)	Band*Pop (MHz*m)	# of Bidders	min ($)	max ($)	# of Winners	Sum Win ($)	min ($)	max ($)
A	176	BEA	12	309	80	7	580268	33	3961174	20	580268
B	734	CMA	12	194	200	1	892400	87	9143993	15	892400
C	15	REA	22	1280	26	13	4713823	4	4748319	550	1625930
D	1	NWA	10	2856	1	472042	472042	1	472042	472042	472042
E	176	BEA	6	154	50	3	224988	5	1266892	17	224988

				Bid			Win
License Name	Market Area	Bandwidth (MHz)	Band*Pop (MHz*m)	# of Bidders	min ($)	max ($)	Win ($)
Mississippi Valley	REA	22	689	5	63932	1625930	1625930
Great Lakes	REA	22	1280	4	171433	1109715	1109715
Chicago, IL	CMA	12	97	6	38223	892400	892400
New York-Newark	CMA	12	194	9	59435	884703	884703
Central	REA	22	888	4	100035	723228	723228
LA-Riverside-Orange	BEA	12	216	6	33044	580268	580268
Northeast	REA	22	1101	3	324585	604624	502774
Los Angeles-Anaheim	CMA	12	187	13	26874	483981	483981
Nationwide	NWA	10	2856	1	472042	472042	472042
NYC-Long Is	BEA	12	309	4	83212	429356	429356

Note: 1) All monetary terms are in thousands of dollars. 2) Block C has 15 licenses: REA1-12, Pkg Atlantic, Pkg Pacific, Pkg 50 States, but there is no bidder for Pkg Pacific,and there is no winner for Pkg Atlantic, Pkg 50 States, REA009 and REA011.

winning regional licenses covering the continental United States and Hawaii (98% of U.S. population).

3.2 Heterogeneous Bidders

We assign bidders into different quartiles based on the size of their upfront payment.[21] Table 2 reports the bidding behaviors and winnings of these quartiles as well as top ten winners. We can see top bidders play the most important role in this auction. Top 25% bidders (53 in total) make the majority of bidding and win 984 out of 1,099 in total available licenses (that is 90% of the licenses). Their gross bids are almost equal to gross bids.[22] AT&T wins more licenses in terms of numbers but Cellco (doing business as Verizon Wireless, referred as Verizon henceforth) wins larger and high value licenses. Many of these winners are industry veterans, including cellphone carriers (for example AT&T, Verizon, and Cellular South), cable companies (for example, Cox), telecommunications firms providing Internet, TV and phone bundles (for example, Frontier), and Internet giant (for example, Google). One thing to note is that many bidders and some winners do not currently offer cellphone services. In other words, they become potential entrants into the industry after acquiring licenses.

Because of the distinct differences in bidding patterns, we later separate bidders into three groups: large, medium and small. Large bidders include just AT&T and Verizon, which are incumbent cellphone carriers with national footprint. Small bidders are bidders that qualifies for FCC's Designated Entity discount – a steep discount (15% and 25% in Auction 73) to encourage small or disadvantaged bidders to acquire spectrum licenses. Medium-sized bidders are bidders who are neither large nor small, including Mobility Spectrum LLC., Airwaves Inc., Qualcomm Inc., MetroPCS 700 MHz, Alltel Corporation, Frontier Wireless and many more.

3.3 Suggestive Evidence for License Complementarity

Figure 1 reports final license allocations of Block A, B, C and E. As shown in the figure, a bidder often wins a cluster of licenses that are located close together. This suggest complementarity across locations. Some bidders, such as Verizon in Block C and Frontier in Block E, aim for nation wide complementarity, while others, such as Cellular South (the dark blue

---

[21] Upfront payment determines the initial eligibility (maximum number of bidding units) of the bidders and is usually positively correlated with the size of the bidder.

[22] Gross bids reported in this table include Qualcomm's provisionally winning bid in Block D, which does not meet FCC's reserve price so is not counted in FCC reported final outcome.

Table 2: Bidders and Winners

		Bid			Win				
Firm Tier	Up.Pay ($)	# Lic.	min ($)	max ($)	# Lic.	Band*Pop (MHz*m)	Sum Win bids ($)	min ($)	max ($)
1st quartile	54212	31298	1	4713823	984	17531	19500000	17	1625930
2nd quartile	437	3148	4	19138	64	107	46955	15	8469
3rd quartile	105	1150	6	8055	24	36	26554	45	8055
4th quartile	30	822	1	2081	18	15	4430	20	793

		Bid			Win				
Firm Name	Up.Pay ($)	# Lic.	min ($)	max ($)	# Lic.	Band*Pop (MHz*m)	Sum Win bids ($)	min ($)	max ($)
AT&T	500000	6052	1	884703	227	2110	6636658	190	884703
Frontier	115253	2269	16	220188	168	1303	711871	51	62656
King Street	97000	3021	9	933360	152	487	400638	38	60918
Verizon Wireless	885000	3783	11	1625930	109	8508	9363160	107	1625930
CenturyTel	25000	2571	9	22151	69	212	148964	93	21928
Triad 700	57000	1296	15	80246	36	186	22694	46	3124
Cavalier	42000	1988	13	66872	35	322	61803	36	7811
Cellular South	29634	580	7	241365	24	180	191533	194	49201
Cox Wireless	36000	638	10	147893	22	248	304633	619	84119
David Miller	2250	384	10	4073	16	40	7812	32	4073

Note: 1) All monetary terms are in thousands of dollars. 2) Band*Pop represents frequency bandwidth times population associated with the license, measured by MHz times millions in population.

area in Block A, covering the majority of Tennessee, Mississippi and Alabama) and King Street (doing business as US Cellular, covering quite a few of the eastern and Midwestern states), aim for regional complementarity. Appendix 1) shows evidence that bidders bid on and stop bidding on multiple licenses at the same time, which provides strong support for the existence of complementarity across different licenses.

3.4 Stylized Facts of Bidder Behaviors

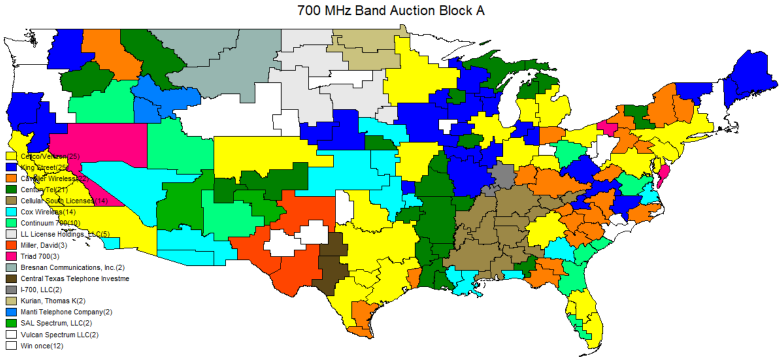
In this section, we establish four stylized facts of bidder behaviors that we observe in data. These stylized facts help us to set up the most parsimonious model to capture a very complicated bidding process.

1) Bidders Bid Minimum Acceptable Bids

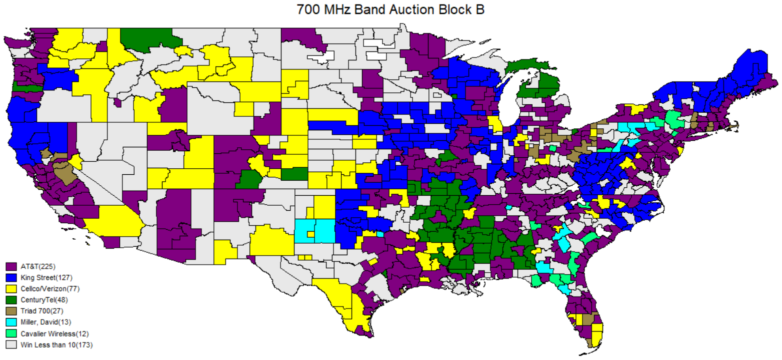
In spectrum auctions, bidding is a discrete choice rather than a continuous choice. In each round of bidding, a bidder can choose from a limited number of possible bids in the (web-based) bidding system. The minimum of these bids is called Minimum Acceptable Bid (MAB), which is determined by FCC. The MAB of a license in a new round equals to

Figure 1: Final License Allocations of Block A, B, C and E

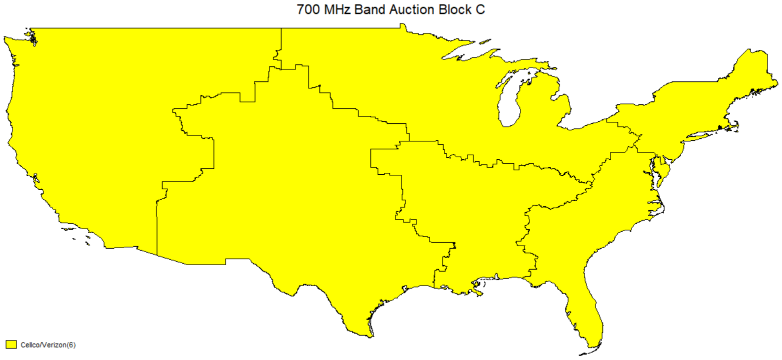
(a) Block A



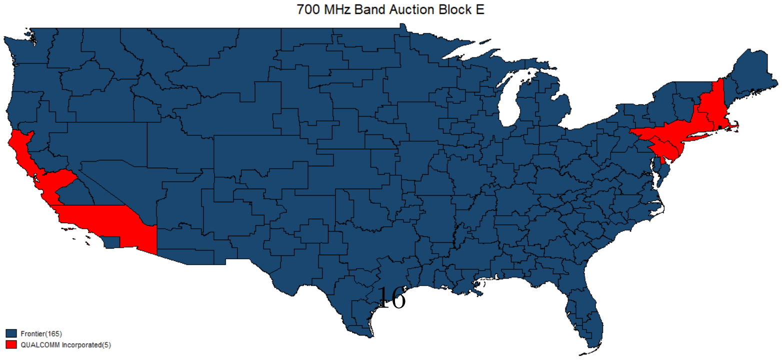
(b) Block B



(c) Block C



(d) Block E



the provisionally winning bid (determined in the last round) plus an increment which may decrease over time. At the beginning of the auction, increment in MAB is around 20% of the provisionally winning bid. At the end of the auction, increment in MAB is usually less than 10% of the provisionally winning bid. For Auction 73 Block C licenses, there are 3 amounts per license or 1 bid amount per package. For other licenses, only one bid amount, the MAB is available. FCC sets this rule because FCC believes that bidders may use jump bids as means of signaling to other bidders. If there are no new bidders placing bid on a license, the MAB of a license may slightly reduce.

From Auction 73's bidding data, we observed that over 99.8% of the bids equals to MAB for an average round. Only 9 bids succeed the MAB, which all occurred before round 17. "Small Ventures USA, L.P." placed 8 of them, and "Copper Valley Wireless, Inc." placed the other one. Both these bidders are middle size players.[23]

After round 17, all bids equal to the MAB. In other words, jump bids are extremely rare in Auction 73. When several bidders place the same bid, a random number is used to determine the provisionally winning bids of the license.

2) Bidders Gradually Drop out of the Bidding Process

Auction 73 went through 261 rounds in total. As we observe from Table 3, this is a "chaotic-initial-rounds-and-then-dust-settling-down" process, as typical of a FCC spectrum auction. In round 1 to 20, the bid-to-bidders ratio is over 100, but this ratio goes down to 50 in round 20 to 40, and then stabilizes at 30 to 40 in later rounds. As shown in the "win" panel of Table 3, most winnings as well as winnings of most high-value licenses are determined in round 21 to 40. Auctions that drag on to over 100 rounds are, in fact, not high-value licenses. In the last row of Table 3, the mean bids are only \$1.2 million and the mean winning bids are only \$2.4 million, both the lowest among all six rows. Overall, bidders participated eagerly in first rounds and gradually dropped out of race round by around, after most winnings are determined by round 40. There is no evidence of bidders waiting strategically until last rounds and coming in to snipe the licenses they are after.

3) Bidders Bid Straightforwardly

Eyeballing the bidding data, we find strong evidence that bidders bid straightforwardly. Though the auction lasts for 261 rounds. The average duration of bids (last bid round - first bid round) is 22 rounds. The vast majority of bidders (88%) bid for fewer than 50 rounds

---

[23] Their upfront payments are \$700,000 and \$528,000, respectively

Table 3: Bidding Statistics Round by Round

	Bid						Win			
Round	# active bidders	# new bids	# active licenses	min ($)	mean ($)	max ($)	# licenses won	min ($)	mean ($)	max ($)
1 to 20	207	21382	1057	1	7111	4713823	86	42	38441	580268
21 to 40	125	6699	960	5	8981	1625930	535	18	27044	1625930
41 to 60	68	3041	325	4	3045	224988	92	32	7699	224988
61 to 80	57	1849	253	4	1827	146963	96	36	2984	62656
81 to 100	46	1434	177	4	1576	154999	91	55	4013	154999
> 100	48	2013	190	6	1255	81613	190	15	2412	81613

Note: All monetary terms are in thousands of dollars.

and stop bidding as MAB increases. In 19% of the bidder-license observations, bidders bid consecutively until they become provisionally winners or until they give up.

We do not see patterns that bidders strategically delay bidding in this auction. The total occurances of "first-time-bidders" after round 20 only counts to about 1000 (bidder-license) observations. (Recall that there are more than 1000 licenses and more than 200 bidders in this auction.) Figure 2 reports, for each round, the percentage of bidder-licenses in which a bidder bids on a license it has not bid on before.[24] Right after round 20, around 3% of bidder-license observations are first time bids. This number reduces dramatically afterwards and becomes negligible after round 50. We suspect these first-time bidders bid either: 1) to satisfy the eligibility requirement after they decide on stop bidding on some licenses; or 2) when some licenses become more valuable as the expected winning probability of adjacent license becomes larger as the auction progresses.

3.5 Stylized Facts and Modeling Choices

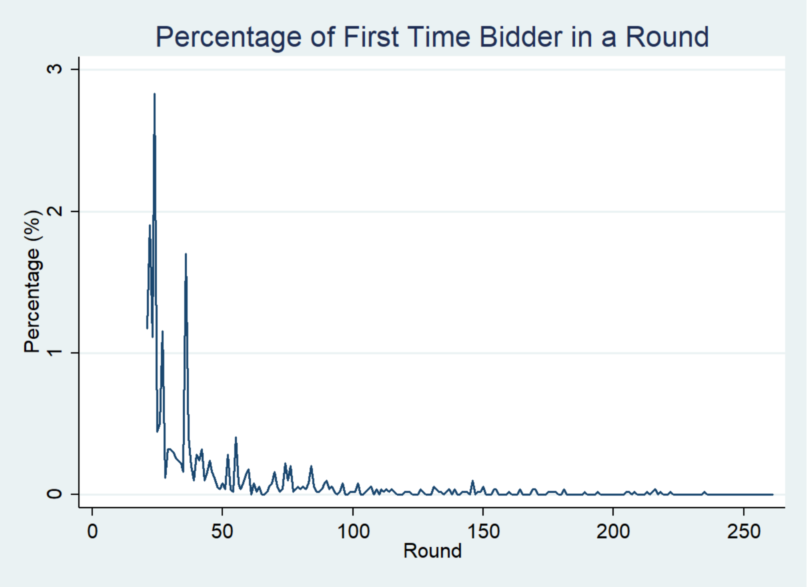
These stylized facts inform us to set up a rich yet tractable model for estimation. First, as the vast majority of bidders bid at MAB, we model an entry problem instead of "name a bid" problem. For any license at any given round, a bidder only needs to decide whether to enter the bidding or not. Second, as rounds progress and bidders gradually drop out from licenses they have bid for, we think this is a process of bidders updating their beliefs at every round. Third and lastly, given the strong evidence that bidders bid straightforwardly, we assume bidders take every round (after the initial rounds[25]) as if this is the last round and act accordingly. That is, we minimize bidders' dynamic considerations and do not consider their "waiting strategically" problem. In fact, under FCC's eligibility and activity requirement,

---

[24] Figure 2 only reports these percentages after round 20.

[25] We drop the initial 20 rounds of the auction from estimation.

Figure 2: Percentage of First Time Bids in Each Round



"waiting" is almost a ruled-out option. In summary, for each round we have a bidder's static entry problem with interdependent markets and the bidder's evolving beliefs about final winning probabilities.

4 Model

We model Auction 73 as an entry game of incomplete information, in which bidders constantly update their beliefs, round by round, of the set of licenses they will win at the conclusion of the auction. Belief updating is the only dynamic feature in our model. This model choice is driven by the features of our setting, which differs from a typical dynamic game (i.e.: Ericson and Pakes (1995)) in three aspects. First, a spectrum auction has finite rounds, however, it is uncertain when the auction will end. The ending round, in fact, is an endogenous outcome of bidder actions. Second, bidders receive no flow payoff during the auction. All pay-offs are realized only at the end of the auction. Third, there are no random shocks in bidders utilities at each round. Bidders' valuation on any sets of licenses remain constant throughout the auction.

4.1 Model Setup and Notation

Notation and Timeline

There are N bidders (each bidder denoted by i) competing for L licenses (each license denoted by l). Rounds are denoted by t with the last round denoted by T.[26] Each license consists of several counties, where counties are the smallest geographic areas in our study. We denote counties by $c \in \mathbf{C}$, and the set of counties covered by license l by $\mathbf{C}_l$.

We now present notation according to the timeline of the game. Right before the auction, bidder i realizes its stand-alone values for all licenses and complementarity between any two licenses. Let v_{il} denote bidder i's stand-alone value of license l, which is the sum of the stand-alone values of all counties in this license (the county-level stand-alone value is denoted by v_{ic}) and complementarity values among these counties. We denote the complementarity index between two licenses l and l' by $\tau(l, l')$. The complementarity index is the same for all bidders although its valuation may be different across different types of bidders. We will discuss its parametrization in Section 5.

At the beginning of a round t, bidder i observes a history h_{it}, which includes the prices and characteristics of all licenses, the number of other bidders in the previous rounds of all licenses, and the bidding history as well as the provisionally winning set of its own.[27] Let P_{lt} denote the minimum acceptable bid of license l in round t. The set of all minimum acceptable bids in round t is $\mathbf{P}_t = \{P_{lt}, l = 1, ..., L\}$. Bidder i also observes the number of other bidders who place a bid on license l in round $t' < t$ (denoted by $N_{-i,l,t'}$). The set of other bidder-license-round characteristics is denoted by $\mathbf{X}_{ilt}$.

Bidders form belief on the set of licenses she can win given history h_{it}. Let $\mathbf{W}_{iT}$ denote the ultimate winning set of bidder i and $Pr(l \in \mathbf{W}_{iT})$ denote bidder i's subjective belief of winning license l at the end of the auction. Let $\mathbf{S}$ denote any set of licenses and $EV_i(\mathbf{S}, Pr(\mathbf{S} \in \mathbf{W}_{iT}))$ denote bidder i's expected value of a set of licenses $\mathbf{S}$ given belief of winning $Pr(\mathbf{S} \in \mathbf{W}_{iT})$. Bidder i selects bidding set $\mathbf{B}_{it}$ to maximize its expected value and FCC reveals the bidder's standing winning set $\mathbf{W}_{it}$ at the end of the round. [28]

---

[26] The ending round, T, is unknown at the beginning of and throughout the auction.

[27] A bidder's provisionally winning set includes the licenses the bidder bid and win in the last round as well as the licenses in the bidder provisional winning set before last round that no new other bidders places a bid on in the last round.

[28] Withdrawals are allowed in the auction but we ignore withdrawal possibilities in our model because bidders rarely withdraw bids. Withdrawn bids consist of less than 1% of the observations.

Information Set

A bidder's information set consists of both public information and private information.

Public information of the bidders include (1) the characteristics of all licenses; (2) minimum acceptable bids of all licenses in the current and previous rounds; (3) complementarity between any two licenses; (4) the number of bidders on all licenses in all previous rounds.

Private information of bidder i include (1) its stand-alone values for all licenses $\{v_{il}, \forall l\}$; (2) its own bidding histories and thus the number of *other* bidders on any licenses in the previous rounds; (3) its provisionally winning set in all previous rounds.

Restrictive Assumptions

We impose some restrictive assumptions. We assume that a bidder's valuation towards a set of licenses is independent of the identities of the winners of the other licenses. Under this assumption, the auction we study is a private value auction. We assume that there is no complementarity or competition across blocks.[29] Moreover, we allow for only pairwise complementarity effect. Most importantly, we assume bidders bid "straightforwardly". Straightforward bidding means that bidders bid as if their current bids in the auction are going to set the final prices. In the simultaneous multiple round auction, that means bidding for precisely the licenses a bidder would want at the current MAB but refraining from bidding on licenses for which a bidder is already the provisional winner.

4.2 Belief Formation

Since there are complementarity between licenses, a bidder's valuation towards a license depends on not only the stand-alone value of the license but also on the set of other licenses she can win. Moreover, bidders do not know exactly the set of licenses they can win *ex-ante*. The ultimate winning set of the bidder i: $\mathbf{W}_{iT}$ is revealed only after the auction concludes. A bidder form beliefs on the probability of winning a set of licenses at the beginning of the auction and updates its beliefs round-by-round during the auction.

It is difficult to obtain the winning belief over a set of licenses as each bidder has 2^L possible ultimate winning set. To simplify our analysis, we impose the following conditional independent assumption:

---

[29] There may exist complementarity or competition effects across blocks. A bidder may want to acquire additional bandwidth, auctioned off in a different block, of the same geographic market. A bidder may also worry about competing against another bidder who acquired bandwidth in another block of the auction after the auction concludes.

Assumption 1 ***CI*** *Conditional on history* h_{it}*, bidder i's subjective belief of winning two licenses are independent:* $Pr(l \in \boldsymbol{W}_{iT} \text{ and } l' \in \boldsymbol{W}_{iT}|h_{it}) = Pr(l \in \boldsymbol{W}_{iT}|h_{it}) \times Pr(l' \in \boldsymbol{W}_{iT}|h_{it})$

This assumes that all complementarity effect in winning beliefs have been revealed in the history. Conditional on history, bidder's belief over winning two licenses are independent. Note we assume away any unobservable heterogeneity at license- or license-set level that are not absorbed by history.

Below, we specify bidder i's belief in winning license l in the auction. As bidders bid straightforwardly, that is, they bid as if the next round is the last round of the auction, bidder i is only able to win license l if she bids on license l or has been the provisional winner of license l. A bidder's belief is different in three cases: (1) license l is in bidder i's bidding set $\mathbf{B}_{it}$; (2) license l is not in bidder i's bidding set $\mathbf{B}_{it}$ but in bidder i's provisional winning set $\mathbf{W}_{it-1}$ in the last round; (3) license l is not in bidder l's bidding set $\mathbf{B}_{it}$ or last round standing winning set $\mathbf{W}_{it-1}$. Bidder i's belief of winning license l is specified as follows:

$$Pr(l \in \mathbf{W}_{iT}|h_{it}) = \begin{cases} Pr(l \in \mathbf{W}_{iT}|l \in \mathbf{B}_{it}, h_{it}) & \text{if } l \in \mathbf{B}_{it} \\ Pr(l \in \mathbf{W}_{iT}|l \in \mathbf{W}_{it-1}\backslash\mathbf{B}_{it}, h_{it}) & \text{if } l \in \mathbf{W}_{it-1}\backslash\mathbf{B}_{it} \\ 0 & \text{if } l \notin (\mathbf{W}_{it-1}\bigcup\mathbf{B}_{it}) \end{cases} \tag{1}$$

where h_{it} denotes the history of bidder i in round t. For notation simplicity, we denote $Pr(l \in \mathbf{W}_{iT}|h_{it})$ or $Pr(\mathbf{S} \in \mathbf{W}_{iT}|h_{it})$ as $Pr(\mathbf{W}_{iT})$ afterwards.

The auction is not a first order Markov process. Bidder's belief of winning licenses may depend on the entire history, which results in enormous number of explanatory variables. We discuss how we reduce the dimensionality of explanatory variables when we estimate beliefs in section 5.

4.3 Stand-alone Value, Complementarity and Expected Payoff

This subsection discusses the stand-alone value, complementarity across licenses, and a bidder's expected value of a set of licenses.

Expected Value of Any Set S

Let $EV_i(\mathbf{S}, Pr(\mathbf{W}_{iT}))$ denote bidder i's expected value of a set of licenses in the set $\mathbf{S}$ with its belief over winning all licenses $Pr(\mathbf{W}_{iT})$. Bidder i's stand-alone value of license l is denoted by v_{il}. Given its belief of $Pr(\mathbf{W}_{iT})$, the bidder's expected value if S equals to the sum of the expected stand-alone values of the licenses in S and the expected complementarity effect among these licenses.

$$\begin{aligned} EV_i(\mathbf{S}, Pr(\mathbf{W}_{iT})) &= \sum_{l \in \mathbf{S}} v_{il} \times Pr(l \in \mathbf{W}_{iT}) + \frac{1}{2}\beta_i \sum_{l \in \mathbf{S}} \sum_{l' \in (\mathbf{S} \backslash l)} \tau(l, l') \times Pr(l \in \mathbf{W}_{iT}, l' \in \mathbf{W}_{iT}) \\ &= \sum_{l \in \mathbf{S}} v_{il} \times Pr(l \in \mathbf{W}_{iT}) + \frac{1}{2}\beta_i \sum_{l \in \mathbf{S}} \sum_{l' \in (\mathbf{S} \backslash l)} \tau(l, l') \times Pr(l \in \mathbf{W}_{iT}) \times Pr(l' \in \mathbf{W}_{iT}) \end{aligned} \tag{2}$$

where β_i denotes the complementarity value from a unit of complementarity index. This complementarity coefficient, β_i, is the key primitive parameter we will estimate using this model. We assume $\beta_i \geq 0$ and we allow different types of bidders to have different complementarity coefficients.

Marginal Contribution of Any License l

The marginal contribution of a license l towards any collection of licenses $\mathbf{S}$ given bidder i's beliefs $Pr(\mathbf{W}_{iT})$ is defined as the difference between the expected value of a set of licenses in $(\mathbf{S} \cup l)$ and a set of licenses in $(\mathbf{S} \backslash l)$.

$$\begin{aligned} \Delta EV_i(l, \mathbf{S}, Pr(\mathbf{W}_{iT})) &= EV_i(\mathbf{S} \cup l, Pr(\mathbf{W}_{iT})) - EV_i(\mathbf{S} \backslash l, Pr(\mathbf{W}_{iT})) \\ &= [v_{il} + \beta_i \sum_{l' \in (\mathbf{S} \backslash l)} \tau(l, l') \times Pr(l' \in \mathbf{W}_{iT})] \times Pr(l \in \mathbf{W}_{iT}) \end{aligned} \tag{3}$$

Expected Payoff of Bidding Set B

A bidder's expected payoff from a set of licenses is the difference between its expected value of the set of licenses and its expected payment. In round t, bidder i observes its provisional winning set $\mathbf{W}_{it-1}$ and a vector $\mathbf{P}_t = \{P_{lt}, l = 1, ..., L\}$ of minimum acceptable bids for each license, forms belief over the probability of winning the set $\mathbf{W}_{it-1} \cup \mathbf{B}_{it}$, and selects a set of licenses $\mathbf{B}_{it}$ to maximize its expected payoff

$$\begin{aligned} &\pi_{it}(\mathbf{B}_{it}, \mathbf{P}_t, Pr(\mathbf{W}_{iT}) | \mathbf{W}_{it-1}) = EV_i(\mathbf{W}_{it-1} \cup \mathbf{B}_{it}, Pr(\mathbf{W}_{iT})) \\ &- \sum_{l \in \mathbf{W}_{it-1} \cup \mathbf{B}_{it}} [P_{it} \times Pr(l \in \mathbf{W}_{iT} | l \in \mathbf{B}_{it}) + P_{i\tilde{t}} \times Pr(l \in \mathbf{W}_{iT} | l \in \mathbf{W}_{it-1} \backslash \mathbf{B}_{it})] \end{aligned} \tag{4}$$

where bidder i's expected payoff of a bidding set $\mathbf{B}_{it}$ in round t equals to its expected value from the set of licenses bidder i can win $(\mathbf{W}_{it-1} \cup \mathbf{B}_{it})$ minus the expected payment. If i bids on a license, she expects to pay the minimum acceptable bid in round t: P_{it}; while if a license is in the provisional winning set of bidder i, she expects to pay the minimum acceptable bid in the round when bidder i became the provisional winner of license l: $P_{i\tilde{t}}$. Note Round $\tilde{t}$ is the round bidder i becomes the provisional winner license l.

Bidders' Decisions

In this model, a license's stand-alone values and the complementarity across licenses are constant over time. Bidders update their beliefs of winning probabilities, observe changes in minimum acceptable bids and make bidding decisions in each round. Bidder i decides on $\mathbf{B}_{it}$ to maximize the **expected** payoff of this set of licenses plus possible winnings in its provisional winning set. A bidder may want to bid on a license for which it has higher probability of winning rather than another license for which its valuation is higher but the expected winning probability is lower.

4.4 Bayesian Nash Equilibrium

The equilibrium concept in this incomplete information game is Bayesian Nash Equilibrium. In each round t, bidder i chooses a set of licenses $\mathbf{B}_{it}$ to maximize its expected payoff. A strategy function of bidder i is denoted by $\sigma_i(\mathbf{v}_i, \tau_i(.), Pr(\mathbf{W}_{iT}))$, where $\mathbf{v}_i$ is the set of stand-alone values of all licenses, $\tau_i(.)$ is the set of complementarity indices between all pairs of licenses, and $Pr(\mathbf{W}_{iT})$ is the bidder's belief of winning any set of licenses. The strategy function maps the private information $\mathbf{v}_i$, common state variables $\tau_i(.)$ and beliefs $Pr(\mathbf{W}_{iT})$ into a set of binary choices. We define a Bayesian-Nash equilibrium as follows:

Definition 1 *A Bayesian-Nash Equilibrium consists of* $\sigma_i^*(\boldsymbol{v}_i, \tau_i(.), Pr^*(\boldsymbol{W}_{iT}))$ *and* $Pr^*(\boldsymbol{W}_{iT})$ *such that for each i,* $\sigma_i^*(\boldsymbol{v}_i, \tau_i(.), Pr^*(\boldsymbol{W}_{iT})) = \boldsymbol{B}_{it}$ *if and only if for any* $\boldsymbol{B}'_{it}$,

$$\pi_{it}(\boldsymbol{B}_{it}, \boldsymbol{P}_t, Pr^*(\boldsymbol{W}_{iT})|\boldsymbol{W}_{it-1}) \geqslant \pi_{it}(\boldsymbol{B}'_{it}, \boldsymbol{P}_t, Pr^*(\boldsymbol{W}_{iT})|\boldsymbol{W}_{it-1}) \tag{5}$$

and if all bidders use strategy $\sigma_i^*(\boldsymbol{v}_i, \tau_i(.), Pr^*(\boldsymbol{W}_{iT}))$, *a bidder' probability of winning a license in the auction is the same as its belief* $Pr^*(\boldsymbol{W}_{iT})$.

According to Brouwer's Theorem, there is at least one Bayesian Nash Equilibrium.

The set of possible bidding sets is enormous. In our case with N bidders and L licenses, in each round a bidder may place 2^L different combinatorial bids and the total number of possible bids for all bidders are $2^{L\times N}$.[30] For a game with state space such highly-dimensional, it is not possible to compute for an equilibrium. Furthermore, there may be multiple equilibria.

---

[30] This is assuming that bidders are not constrained by their eligibility.

To estimate the model, we specify two behavior assumptions, which are necessary conditions of all Bayesian Nash Equilibria in this game and we exploit these behavioral assumptions to estimate the primitives of the model.

4.5 Behavior Assumptions

Suppose in data we observe that a bidder bids for a set of licenses $\mathbf{B}_{it}$ in round t. As the bidder thinks every round is the last round, it reveals its true preferences. Therefore, the expected payoffs from bidding on $\mathbf{B}_{it}$ should be greater than or equal to zero.

Behavior Assumption 1 ***(BA1)*** *If* $l \in \boldsymbol{B}_{it}$, $\Delta V_i(l, \boldsymbol{W}_{it-1} \cup \boldsymbol{B}_{it}, Pr(\boldsymbol{W}_{iT})) \geq P_{lt} \times Pr(l \in \boldsymbol{W}_{iT})$

BA1 can be simplified to

$$v_{il} + \beta_i \times \sum_{l' \in (\mathbf{W}_{it-1} \cup \mathbf{B}_{it} \backslash l)} \tau(l, l') \times Pr(l' \in \mathbf{W}_{iT}) \quad \geq \quad P_{lt} \tag{6}$$

BA1 states that, if a bidder bids on license l, the marginal contribution of license l is higher than the minimum acceptable bid of this license. **BA1** is a necessary condition for a bidder's optimal behavior of this game, even if we allow more strategic bidding behavior than straightforward bidding.

Similarly, we construct another behavior assumption to capture the revealed preferences of a bidder not bidding on a license:

Behavior Assumption 2 ***(BA2)*** *If* $l \notin \boldsymbol{B}_{it}$ *and* $l \notin \boldsymbol{W}_{it-1}$ *but* l *is under bidder* i*'s eligibility constraint,* $\Delta V_i(l, \boldsymbol{W}_{it-1} \cup \boldsymbol{B}_{it}, Pr(\boldsymbol{W}_{iT})) \leq P_{lt} \times Pr(l \in \boldsymbol{W}_{iT})$

BA2 can be simplified to

$$v_{il} + \beta_i \times \sum_{l' \in (\mathbf{W}_{it-1} \cup \mathbf{B}_{it} \backslash l)} \tau(l, l') \times Pr(l' \in \mathbf{W}_{iT}) \quad \leq \quad P_{lt} \tag{7}$$

BA2 states that, when a bidder does not bid on license l although l is within its eligibility constraint, the marginal contribution of license l is lower than the minimum acceptable bid of license l in this round.

For **BA2** to hold, a bidder cannot strategically delay bidding on a license. Under our "straightforward" bidding assumption, every period is the last period so there is no room for strategic delay. To ensure this assumption hold, we discard the first 20 rounds of bidding from estimation. We also argue that FCC's activity requirement is very effective in alleviating this concern (as shown by evidence presented in section 3.4).

These two behavior assumptions can explain why a bidder starts to bid on a new license that it has not bid before while the minimum acceptable bid monotonically increases, and also why a bidder eventually stops bidding. At early rounds, a bidder assigns low probabilities of winning licenses, so the expected complementarity of any license is small. When a bidder is more confident that it will win some licenses, it will start to bid on other licenses to gain complementarity. Eventually, it may find the price of a license too high and gives up bidding.

5 Identification, Estimation and Results

In this section, we first discuss the identification of the complementarity coefficient β_i and the stand-alone values of licenses. We then estimate the primitives of the model in the order of: 1) bidders' beliefs of winning probabilities, 2) the complementarity coefficient β_i, and 3) the stand-alone values of licenses as a function of license and bidder characteristics.

We estimate the above primitives sequentially instead of estimating them simultaneously. The advantage of sequential estimation is that any possible misspecification of one primitive will not affect the estimation of another. For example, we do not need to impose any parametric specification assumptions on stand-alone values in the estimation of complementarity effect. This way the complementarity effect, the main focus of this study, is more robust to potential model misspecification. The disadvantage of sequential estimation is that the standard errors of estimates in the previous step may affect the standard errors of those in later steps. To solve this problem, we use bootstrap to correct the standard errors in the estimation of stand-alone values of licenses.

5.1 Identification

Identification of Complementarity Effect

We make use of changes in a bidder's bidding decisions on a license over different rounds to identify the complementarity effect. For bidder i on a license l, the stand-alone value of the license v_{il} and the complementarity between license l and any other license l' remains the same throughout the auction. The minimum acceptable bid P_{lt}, a bidder's belief of winning other licenses $Pr(l' \in \mathbf{W}_{it}), \forall l' \neq l$, and most importantly, a bidder's provisional winning set at each round change over time. The latter two elements determine the expected contribution of complementarity of a license to a bidder in each round. Changes in a bidder's bidding decisions on a given license result from changes in the minimum acceptable bids and the bidder's expected contribution of complementarity by the license. Since we observe the minimum acceptable bids in data, we can exploit the bidder's bidding decisions on a license given prices over different rounds to bound the complementarity effect.[31]

When we observe a bidder starts bidding on a license it has not bid on in earlier rounds, this is because the increase in price is lower than the increase in the marginal contribution of the license. Since the stand-alone value of the license remains the same, this start-bidding decision at a given price generates a lower bound for the expected complementarity effect. When we observe a bidder stops bidding on a license it has been bidding on, this is because the increase in license price is higher than the increase in the marginal contribution of the license. Similarly, the stop-bidding decision at a given price generates an upper bound for the expected complementarity effect. The upper and lower bounds of the complementarity effect are independent of the stand-alone value of licenses, and therefore we can separately identify the complementarity effect from the stand-alone value of licenses.

Identification of Stand-alone Values

We identify the stand-alone value of a license from a bidder's bidding decisions across different licenses. If a bidder chooses to bid on a license, the marginal contribution of this license must be greater than the license's minimum acceptable bid. The "revealed preference" in this action generates a lower bound for the stand-alone value (as the complementarity effect is already separately identified). On the contrary, if the bidder has eligibility to bid on a license but chooses not to do so, the "revealed preference" in this action generates an upper bound for the stand-alone value.

We will parametrize the stand-alone value of a license as a function of license- and bidder-level characteristics. As different licenses and bidders have different characteristics, a bidder's different bidding decisions across licenses in a given round identifies how these characteristics enter our parametrization. Basically, if a bidder bid on license l instead of on license l', the marginal contribution of license l towards a bidder's expected payoff is higher than that of

---

[31] In our model, marginal contribution of a license is shifted by each bidder's current provisional winner set, which are excluded from the utility of another license. Fox and Lazzati (2017) and Gentzkow (2007) talk about using 'variables that can be excluded a priori from the utility of one or more goods.' In Gentzkow (2007), examples are "whether consumers have Internet access at work or a fast connection athome, which shift the utility of the onlineedition without affecting the utility of theprint edition". Fox and Lazzati (2017) is more about non-parametric identification.

license l'. As the complementarity effect is already identified from the previous step, the stand-alone values of different licenses can be separated from the marginal contributions of these licenses. In Appendix 2), we state a behavioral assumption **BA3**, which specify the necessary conditions in a bidder's actions on different licences. We prove that our **BA1** and **BA2** are sufficient conditions for **BA3**. Therefore, our **BA1** and **BA2** are sufficient for identification and estimation of this model.

5.2 Estimation of Belief

5.2.1 Specification of Bidder Belief

This subsection discusses estimation of bidders' beliefs of winning a license. In round t, bidder i's belief of winning license l may depend on the entire history. We use the following parametric specification, allowing bidder belief only depending on history up to the last period, but our specification can easily go back more periods.[32]

If the bidder places a bid in round t, bidder i's belief of winning license l is

$$Pr(l \in \mathbf{W}_{iT} | l \in \mathbf{B}_{it}) = \Phi(\alpha_0^B + \alpha_1^B N_{-i,l,t-1} + \theta_1^B \sum_{l' \neq l} \tau(l', l) 1[l' \in W_{i,t-1}] + \mathbf{X}_{ilt}^B \gamma^B) \quad (8)$$

where $N_{-i,l,t-1}$ is the number of other bidders that bid on license l plus the provisional winner of license l in round $t-1$. $\sum_{l' \neq l} \tau(l', l) 1[l' \in W_{i,t-1}]$ is the contribution of license l to the complementarity effect in the provisional winning set of bidder i. θ_1^B measures how bidder i's belief over winning license l changes when the contribution of license l to the complementarity effect changes. A Bidder may be more likely to win a license if it can win other licenses that have large complementarity with the license; θ_1^B captures this effect. The set of bidder-license-round characteristics X_{ilt}^B includes the number of rounds, the log upfront payment, the log number of bids placed by the bidder in a round, the tower density a bidder has in the license and bidder fixed effects. Lastly, $\Phi(.)$ is the Normal density function.[33]

If the bidder does not bid on license l but it is a provisional winner of license l in round $t-1$, its belief of winning the license is (using the same notation as the previous equation):

---

[32] The large number of possible explanatory variables renders it infeasible to use non-parametric estimation.

[33] The tower density variable is defined as $\ln(\frac{Tower_{il}}{Area_l} + 1)$, where $Tower_{il}$ is the number of cellphone towers bidder i has (by a certain date) and $Area_l$ is the fraction of license $l's$ area in the United States. Appendix 3) explains how we construct the cellphone tower variable using FCC cellphone tower registration database.

$$Pr(l \in \mathbf{W}_{iT} | l \in \mathbf{W}_{it-1}) = \Phi(\alpha_0^W + \alpha_1^W N_{-i,l,t-1} + \theta_1^W \sum_{l' \neq l} \tau(l', l) 1[l' \in W_{i,t-1}] + \mathbf{X}_{ilt}^W \gamma^W) \quad (9)$$

where $N_{-i,l,t-1}$ is the number of other bidders that bid on license l in round $t-1$, and the set of bidder-license-round characteristics X_{ilt}^W includes X_{ilt}^B and the number of rounds bidder i is the provisional winner of license l .

5.2.2 Estimates of Bidder Belief

Table 4 reports estimates of bidder belief. Column (1) reports the estimates of the probability of winning a license if a bidder bids on a license. Column (2) reports the marginal effect at covariates' mean for the estimates in Column (1). Column (3) reports the estimates of the probability of winning a license if the bidder is a provisional winner of the license. Column (4) reports the marginal effect at covariates' mean for the estimates in Column (3). All marginal effects reported in this table are statistically significant.

For an active bidder on a license, when the number of round increases by one, its winning probability increases by 0.2%. When the bidder's upfront payment increase by one percent, its winning probability increases by 5.5%. When the number of competitors in the last round increases by one, its winning probability decreases by 10.1%. When the complementarity index between the focal license and the bidder's provisional winning set in the last round increases by 0.01, its winning probability increases by 1.847%. When the log of tower density of a bidder on a license increases by 1, its winning probability increases by 1.5%. Comparing to medium bidders, ceteris paribus, large (small) bidders are 26.2% (10.6%) less likely to win a license.

Results for provisional winners are (mostly) qualitatively similar but quantitatively different. For a provisional winning bidder on a license, when the number of round increases by one, its winning probability increases by 0.01%. When the bidder's upfront payment increases by one percent, its winning probability increases by 0.3%. When the number of competitors in the last round increases by one, its winning probability decreases by 2.2%. When the complementarity index between this license and bidder's provisional winning set in the last round increase by 0.01, its winning probability increases by 1.2%. When the log of tower density doubles, its winning probability increases by 0.1%. Moreover, when the total number of rounds that the bidder is the provisional winner increases by one round, the probability that the bidder will win the license increases by 0.03%. Comparing to medium

Table 4: Belief Estimation Result

	If Bidder Bids		If Bidder Provisional Wins	
	(1) Estimate	(2) Marg. Effect	(3) Estimate	(4) Marg. Effect
# Round	.004***	.002***	.005***	.0001***
	(.000)	(.000)	(.000)	(.000)
ln(upfront payment)	.142***	.055***	.117***	.003***
	(.005)	(.002)	(.002)	(.000)
# Competitors last round	-.259***	-.101***	-.831***	-.022***
	(.026)	(.01)	(.028)	(.001)
Complementarity contribution	4.737***	1.847***	.435***	.012***
	(.760)	(.296)	(.061)	(.002)
# $\ln(\frac{Tower_{il}}{Area_l} + 1)$	.038***	.015***	.042***	.001***
	(.005)	(.002)	(.002)	(.000)
# Round provisional win			.014***	.0003***
			(.000)	(.000)
Large Bidder	-.755***	-.262***	-.514***	-.018***
	(.046)	(.013)	(.017)	(.001)
Small Bidder	-.274***	-.106***	-.219***	-.006***
	(.024)	(.009)	(.011)	(.000)
Constant	-2.371***		-1.597***	
	(.096)		(.036)	
Pseudo. R^2	0.0816		0.3263	
# obs (bidder-license-round)	14,984	14,984	259,295	259,295

Note: Bidder fixed effects are included in all specifications.

bidders, ceteris paribus, large (small) bidders are 1.8% (0.6%) less likely to win a license.

Overall, the most notable pattern in Table 4 is the strong competition effect and the even stronger complementarity effect. The presence of competition lowers the winning probability while the presence of complementarity increases the winning probability. A bidder should consider all factors determining winnings when considering whether to bid on a license.

5.3 Estimation of Complementarity

5.3.1 Specification of Complementarity Index

This subsection discusses the construction of complementarity index $\tau(l, l')$.

County Complementarity Index

As described in Section 2, different blocks have different geographic delineation that divides the U.S. and its territories into exclusive market areas. Geographic area definitions in Block A and E are BEAs, in Block B CMA, in Block C REA, and in block D nationwide. To make different blocks comparable, we use county, the greatest common divisor of CMA, BEA and REA, as the smallest geographic area in this study. Every license can be split into a number of whole counties and every county is always entirely and exclusively included in a license.[34]

We follow the specification of complementarity index in Fox and Bajari (2013). Complementarity index between any two counties c and c' with a bandwidth of $Bandwidth_{c,c'}$ is

$$\tau(c, c') = Bandwidth_{c,c'} \left[pop_c \frac{\frac{pop_c pop_{c'}}{dist^{\delta}_{c,c'}}}{\sum_{c'' \in \mathbf{C} \setminus c} \frac{pop_c pop_{c''}}{dist^{\delta}_{c,c''}}} + pop_{c'} \frac{\frac{pop_c pop_{c'}}{dist^{\delta}_{c,c'}}}{\sum_{c'' \in \mathbf{C} \setminus c'} \frac{pop_{c'} pop_{c''}}{dist^{\delta}_{c',c''}}} \right] \tag{10}$$

where pop_c is the fractions of the U.S. population in county c, $dist_{c,c'}$ is the distance between two counties c and c',[35] $\mathbf{C}$ is the set of all counties and $Bandwidth$ is the bandwidth of the block. We set $\delta = 2$ such that our model is close to the "gravity model". A nice property of this index is that a nationwide license with bandwidth of 1 MHz has total complementarity index of one.[36]

---

[34] In contrast, Fox and Bajari (2013) uses Basic Trading Area as the smallest geographic area in their study, which divides the continental United States into 480 mutually exclusive licenses.

[35] The distance between two counties are computed as the distance of the two county centers and measured in kilometers. The minimum distance between two counties is set to be 10 kilometers to avoid small denominators.

[36] In Appendix 4), we compare different methods of constructing the complementarity index and find them to be similar in summary statistics.

License Complementarity Index

Since each license (CMA, BEA or REA) exclusively contains one or more counties, we define complementarity index between any two licenses l and l' as the sum of complementarity indices between the set of counties in l and the set of counties in l'.

$$\tau(l,l') = \sum_{c\in\mathbf{C}_l}\sum_{c'\in\mathbf{C}_{l'}} \tau(c,c') \tag{11}$$

where $\mathbf{C}_l$ is the set of counties within license l.

License Stand-alone Value and Within License Complementarity

The stand-alone value of a CMA, BEA or REA license l consists of: (1) the stand-alone values of all counties within license l; and (2) the complementarity effect between any two counties within license l. Let v_{il} denote the stand-alone value of license l,

$$v_{il} = \sum_{c\in\mathbf{C}_l} v_{ic} + \frac{1}{2}\beta_i \sum_{c\in\mathbf{C}_l}\sum_{c'\in\mathbf{C}_l\backslash c} \tau(c,c') \tag{12}$$

We divide the complementarity index by two to eliminate double counting in the complementarity index between two counties within a license.

Construction of Inequalities

We construct a set of inequalities by making use of **BA1** and **BA2**. For bidder i on license l, the set $\mathbf{BID}_{il}$ denotes the set of rounds bidder i bidding on license l. According to **BA1**, if round $t \in \mathbf{BID}_{il}$, the marginal contribution of the license in round t is greater than the minimum acceptable bid of the license in round t.

$$v_{il} + \beta_i \sum_{l'\in(\mathbf{W}_{it-1}\cup\mathbf{B}_{it}\backslash l)} \tau(l,l') \times Pr(l' \in \mathbf{W}_{iT}) \quad \geq \quad P_{lt} \tag{13}$$

In this inequality, $Pr(l' \in \mathbf{W}_{iT})$ is the "true" belief of the bidder, which is not observable to the econometrician. We replace this "true" belief with the estimated belief $\widehat{Pr}_t(l' \in \mathbf{W}_{iT})$. This way we naturally introduce an error term ε_{ilt}, which is the difference between the expected complementarity contribution of a license under a bidder's "true' belief and that under its estimated belief.

$$\varepsilon_{ilt} = \beta_i(\sum_{l'\in(\mathbf{W}_{it-1}\cup\mathbf{B}_{it}\backslash l)} \tau(l,l') \times Pr(l' \in \mathbf{W}_{iT}) - \sum_{l'\in(\mathbf{W}_{it-1}\cup\mathbf{B}_{it}\backslash l)} \tau(l,l') \times \widehat{Pr}(l' \in \mathbf{W}_{iT}))\tag{14}$$

and inequality (13) becomes:

$$v_{il} + \beta_i SP(ilt) + \varepsilon_{ilt} \geq \quad P_{lt} \tag{15}$$

where $SP(ilt) = \sum_{l' \in (\mathbf{W}_{it-1} \cup \mathbf{B}_{it} \backslash l)} \tau(l,l') \times \widehat{Pr}(l' \in \mathbf{W}_{iT})$ is the expected complementarity contribution of a license under a bidder's estimated belief $\widehat{Pr}(l' \in \mathbf{W}_{iT})$.

Similarly, for bidder i on license l, the set $\mathbf{NOTBID}_{il}$ denotes the set of rounds that bidder i does not bid on license l while bidder i is not the provisional winner of license l and the license's assigned bidding units are under bidder i's unused eligibility. According to **BA2**, if $t' \in \mathbf{NOTBID}_{il}$

$$v_{il} + \beta_i \sum_{l' \in (\mathbf{W}_{it'-1} \cup \mathbf{B}_{it'} \backslash l)} \tau(l,l') \times Pr(l' \in \mathbf{W}_{iT}) \quad \leq \quad P_{lt'} \tag{16}$$

We then replace this "true" belief with the estimated belief $\widehat{Pr}(l' \in \mathbf{W}_{iT})$ and introduce an error term $\varepsilon_{ilt'}$, which is the difference between the expected complementarity contribution of a license under a bidder's "true" belief and that under its estimated belief.

$$v_{il} + \beta_i SP(ilt') + \varepsilon_{ilt'} \quad \leq \quad P_{lt'} \tag{17}$$

Once we have the two sets $\mathbf{BID}_{il}$ and $\mathbf{NOTBID}_{il}$, we match a round t in set $\mathbf{BID}_{il}$ with any round t' in set $\mathbf{NOTBID}_{il}$, sum up two inequalities and eliminate v_{il}:

$$\beta_i \left[SP(ilt) - SP(ilt')\right] + \varepsilon_{ilt} - \varepsilon_{ilt'} \quad \geq \quad P_{lt} - P_{lt'} \tag{18}$$

Define $DS(l,t,t') = SP(ilt) - SP(ilt')$, where $DS(l,t,t')$ is the difference between the expected complementarity contribution of a license under a bidder's estimated belief in round t and that in round t'. We now construct conditional moment inequality:

$$E[\beta_i DS(l,t,t') - P_{lt} + P_{lt'} + \varepsilon_{ilt} - \varepsilon_{ilt'} | SP(ilt), SP(ilt'), Bid_{ilt}] \geq 0 \tag{19}$$

Assumption 2 ***CI2*** *Conditional on the expected complementarity and bidding decisions, errors generated by calculating the expected complementarity contribution of a license using a bidder's estimated belief have zero mean:* $E[\varepsilon_{ilt}|SP(ilt), Bid_{ilt}] = 0$

This assumption means that we do not allow for unobservable heterogeneity that bidders observe but the econometrician do not observe. We can now rewrite the conditional inequality as:

$$E[\beta_i DS(l,t,t') - P_{lt} + P_{lt'} | DS(l,t,t'), Bid_{ilt}] \geq 0 \tag{20}$$

Follow Andrews and Shi (2013), we transform the conditional inequality in Equation 20 to unconditional inequality without losing identification power by properly selecting instruments. For a non-negative instrument $z(DS(l,t,t'))$, we have

$$E\left[z(DS(l,t,t'))[\beta_i DS(l,t,t') - P_{lt} + P_{lt'}]\right] \geq 0 \tag{21}$$

Construction of Criterion Functions

Below we discuss the construction of the criterion functions. All inequalities belong to one of the following four categories:

Cat. 1: When $DS(l,t,t') > 0$ and $P_{lt} - P_{lt'} > 0$, Equation 21 implies $\beta_i \geq \frac{E[z(DS(l,t,t'))(P_{lt}-P_{lt'})]}{E[z(DS(l,t,t'))DS(l,t,t')]}$. This is a lower bound of the complementarity coefficient β_i. If bidder i does not bid on license l when price is low (in round t') but starts to bid on the license when price is high (in round t), the increase in the expected complementarity contribution of this license must be higher than the increase in prices, which generates a lower bound for complementarity.

Cat. 2: When $DS(l,t,t') < 0$ and $P_{lt} - P_{lt'} < 0$, Equation 21 implies $\beta_i \leq \frac{E[z(DS(l,t,t'))(P_{lt}-P_{lt'})]}{E[z(DS(l,t,t'))DS(l,t,t')]}$. This is an upper bound of complementarity effect. If bidder i bids on on license l when price is low (in round t) but stops bidding on this license when price is high (in round t'), the increase in the expected complementarity contribution of this license must be lower than the increase in prices, which generates an upper bound for complementarity.

Cat. 3: When $DS(l,t,t') > 0$ and $P_{lt} - P_{lt'} \leq 0$, Equation 21 implies $\beta_i \geq$ a non-positive number. However, it is uninformative because we assume $\beta_i \geq 0$. So inequalities in Cat 3 do not provide us any new information.

Cat. 4: When $DS(l,t,t') < 0$ and $P_{lt} - P_{lt'} \geq 0$, Equation 21 implies $\beta_i \leq 0$. This contradicts our model because we assume $\beta_i \geq 0$.

To estimate our model, we drop inequalities that either provide no information (Cat. 3) or contradict our model (Cat. 4) but use only inequalities in Cat. 1 and Cat. 2 to estimate the complementarity effect. These two sets of inequalities identify the complementarity coefficient β_i.

We use moment inequality approach to estimate the complementarity effect. We define the following criterion function following Chernozhukov, Hong, and Tamer (2007)

$$Q\left(\beta_i\right) = \sum_{z(DS(l,t,t'))} \min\{z(DS(l,t,t')) \times [\beta_i DS(l,t,t') - P_{lt} + P_{lt'}], 0\}^2 \tag{22}$$

where $\{z(DS(l,t,t'))\}$ is a set of instruments. Following Andrews and Shi (2013), we divide all inequalities to different groups and instrument $z(.)$ is an indicator function whose value equals one if inequalities belong to a group and zero otherwise. The selection of groups are based on the quantiles of $DS(.)$ values. For instance, when we construct instruments for inequalities in Cat. 1, we divide all inequalities to 4, 6, 8, 10, 12 and 14 groups. Suppose we divide inequalities into 4 groups, the first group includes all inequalities in the first quartile and the last group includes those inequalities in the fourth quartile. The instrument $z(.)$ indicates whether an inequality belongs to a group. There will be a total of 108 instruments.[37]

The estimates of complementarity effect minimize the criterion function. The true value of β_i should satisfy all inequalities if we specify the model correctly. If there are multiple (a set of) estimates of β_i that satisfy all inequalities (when $\beta_i = \hat{\beta}_i$, the value of criterion function $= 0$), then we have a set estimate. If there is no estimate that satisfies all inequalities (there is no $\beta_i = \hat{\beta}_i$ such that the value of criterion function $= 0$), we select a $\hat{\beta}_i$ that minimizes the criterion function as a point estimate.

Lastly, to allow for bidder heterogeneity in bidders' valuation of complementarity, we separately estimate the complementarity parameter for large, medium, and smaller bidders.

5.3.2 Estimates of the Complementarity Effect

Estimates of complementarity effect are reported in Table 5. Column (1) reports empirical results for the full sample, Column (2) for large bidders, Column (3) for medium bidders, and Column (4) for small bidders. In the estimation, for the set $\mathbf{NOTBID}_{il}$ (when bidder i does not bid on license l), we only consider licenses whose assigned bidding units are lower than the unused eligibility of the bidder. Appendix 5) reports results from a robustness check, in which we add more restriction for the set $\mathbf{NOTBID}_{il}$: the bidding units of the license need to be higher than additional units required to maintain the eligibility level of the bidder (but still lower than the unused eligibility of the bidder). In this robustness check, we consider the possibility that a small license will not help the bidder to satisfy its activity requirement.

The estimates indicate that, when the complementarity index (of a nationwide license)

---

[37] The total number of groups is $4+6+8+10+12+14 = 54$ for each of the two categories of inequalities.

Table 5: Estimates of Complementarity Effect (in million $)

	(1) Full Sample	(2) Large	(3) Medium	(4) Small
β	194	918	222	120
95% CI	[186,202]	[635,1021]	[214,230]	[117,125]
# LBs	107651	4796	64046	38809
# Ubs	266437	10511	117285	138641

Note: the unit of observation is bidder-license-(round-pairs). For each bidder-license, we match each round where the bidder place a bid with a round where the bidder does not place a bid.

increases by 1 MHz, the value from complementarity increases by $194 million. The bandwidths in Block A, B, C, D, and E are 12,12, 22, 10, and 6 respectively, the total value of complementarity if a bidder bids all licenses in all five blocks is $(12+12+22+10+6)$ times $194 million, which is $12 billion in total and around $2.4 billion per block. Since total biddings are around $19 billion in Auction 73, the total complementarity effect is equivalent to about two thirds of the total bidding. Large bidders value complementarity more than smaller bidders. The value of complementarity in a nationwide license with bandwidth of 1 MHz is worth $918 million to an average large bidder, $222 million to an average medium bidder, and only $120 million an average small bidder.[38]

5.4 Estimation of the Stand-alone Value of a License

5.4.1 Specification of the Stand-alone Value Function

After we obtain estimates of the complementarity effect, we now proceed to estimate bidders' stand-alone values of licenses. We parametrize v_{ic}, bidder i's stand-alone value in county c as a function of license characteristics and bidder characteristics:

$$\begin{aligned} v_{ic} = \{&\theta_1 \ln(\frac{Pop_c}{Area_c}+1) + \theta_2 \ln(\frac{Tower_{ic}}{Area_c}+1) + \theta_3 \ln \text{UpPay}_i + \theta_4 1[Large_i] \\ &+\theta_5 1[Medium_i] + \theta_6 1[Small_i]\} \times Pop_c \times Bandwidth_c + \xi_{ic} \end{aligned} \tag{23}$$

where Pop_c is the fraction of county c's population in the United States, $Area_c$ is the fraction of county c's area in the United States, and $Tower_{ic}$ is the number of bidder i's towers in

---

[38] In Appendix 6) for comparison we report estimation results from a misspecified model, in which a bidder assigns 100% probability of winning every license it bids on or is the provisional winner of.

county c. We use these three variables to construct $\frac{Pop_c}{Area_c}$, the population density of the county c, and $\ln(\frac{Tower_{ic}}{Area_c}+1)$, the tower density of bidder i in county c, in the specification. We further include $\ln \text{UpPay}_i$, the natural log of the upfront payment of bidder i measured by dollars, $1[Large_i]$, $1[Medium_i]$ and $1[Small_i]$ are dummy variables to indicate whether bidder i belongs to the category of large, medium or small bidders. These five variables, interacted with Pop_c and $Bandwidth_c$, determine the stand-alone value of a license.

According to Equation (12), bidder i's stand-alone value for license l includes stand-alone values of all counties within license l and the complementarity effect among counties within license l:

$$\begin{aligned} v_{il} &= \sum_{c\in l}\{\theta_1 \ln(\frac{Pop_c}{Area_c}+1)+\theta_2\ln(\frac{Tower_{ic}}{Area_c}+1)+\theta_3\ln \text{UpPay}_i \\ &\quad +\theta_4 1[Large_i]+\theta_5 1[Medium_i]+\theta_6 1[Small_i]\}\times Pop_c\times Bandwidth_c \\ &\quad +\frac{1}{2}\beta_i\times\sum_{c\in\mathbf{C}_l}\sum_{c'\in\mathbf{C}_l\backslash c}\tau(c,c')+\xi_{il}, \end{aligned} \tag{24}$$

where $\xi_{il}=\sum_{c\in\mathbf{C}_l}\xi_{ic}$ and $\mathbf{C}_l$ is the set of counties within license l.

Estimation of stand-alone values are based on both behavior assumptions: **BA1** generates a lower bound for stand-alone values while **BA2** generates an upper bound for stand-alone values.

If bidder i places a bid on license l in round t, according to Inequality (6), we have

$$1[Bid_{ilt}=1](B_{ilt}(\theta)+\xi_{il}+\varepsilon_{ilt})\geq 0 \tag{25}$$

where

$$\begin{aligned} B_{ilt}(\theta) &= \sum_{c\in\mathbf{C}_l}\{\theta_1 \ln(\frac{Pop_c}{Area_c}+1)+\theta_2\ln(\frac{Tower_{ic}}{Area_c}+1)+\theta_3\ln \text{UpPay}_i \\ &\quad +\theta_4 1[Large_i]+\theta_5 1[Medium_i]+\theta_6 1[Small_i]\}\times Pop_c\times Bandwidth_c \\ &\quad +\frac{1}{2}\beta_i\times\sum_{c\in\mathbf{C}_l}\sum_{c'\in\mathbf{C}_l\backslash c}\tau(c,c') \\ &\quad +\beta_i\times\sum_{l'\in(\mathbf{W}_{it-1}\cup\mathbf{B}_{it}\backslash l)}\tau(l,l')\times\widehat{Pr}_t(l'\in\mathbf{W}_{iT})-P_{it} \end{aligned} \tag{26}$$

represents the parametric component of the expected marginal contribution of license l minus the minimum acceptable bid of license l. Average across all bids bidder i placed on license l

and we have

$$1[Bid_{ilt}=1](\overline{B}_{ilt}^{Bid=1}(\theta)+\xi_{il}+\overline{\varepsilon}_{ilt}^{Bid=1})\geq 0 \tag{27}$$

where $\overline{B}_{ilt}^{Bid=1}(\theta)=\frac{\sum_{ilt}1[Bid_{ilt}=1]B_{ilt}(\theta)}{\sum_{ilt}1[Bid_{ilt}=1]}$ and $\overline{\varepsilon}_{ilt}^{Bid=1}=\frac{\sum_{ilt}1[Bid_{ilt}=1]\varepsilon_{ilt}}{\sum_{ilt}1[Bid_{ilt}=1]}$.

On the other hand, if bidder i does not place a bid on license l in round t, according to Inequality (7), we have

$$-1[Bid_{ilt}=0](\overline{B}_{ilt}^{Bid=0}(\theta)+\xi_{il}+\overline{\varepsilon}_{ilt}^{Bid=0})\geq 0 \tag{28}$$

where $\overline{B}_{ilt}^{Bid=0}(\theta)=\frac{\sum_{ilt}1[Bid_{ilt}=0]B_{ilt}(\theta)}{\sum_{ilt}1[Bid_{ilt}=0]}$ and $\overline{\varepsilon}_{ilt}^{Bid=0}=\frac{\sum_{ilt}1[Bid_{ilt}=0]\varepsilon_{ilt}}{\sum_{ilt}1[Bid_{ilt}=0]}$.

Sum up Inequality (27) and Inequality (28)

$$\begin{aligned}1[Bid_{ilt}=1](\overline{B}_{ilt}^{Bid=1}(\theta)+\xi_{il}+\overline{\varepsilon}_{ilt}^{Bid=1})-1[Bid_{ilt}=0](\overline{B}_{ilt}^{Bid=0}(\theta)+\xi_{il}+\overline{\varepsilon}_{ilt}^{Bid=0})&\\ \geq 0&\end{aligned} \tag{29}$$

We can obtain conditional moment inequality:

$$\begin{aligned}E[1[Bid_{ilt}=1](\overline{B}_{ilt}^{Bid=1}(\theta)+\xi_{il}+\overline{\varepsilon}_{ilt}^{Bid=1})\\ -1[Bid_{ilt}=0](\overline{B}_{ilt}^{Bid=0}(\theta)+\xi_{il}+\overline{\varepsilon}_{ilt}^{Bid=0})|Bid_{ilt},\mathbf{X}_{il}]\geq 0\end{aligned} \tag{30}$$

where $\mathbf{X}_{il}=\{\ln(\frac{Pop_l}{Area_l}+1),\ln(\frac{Tower_{il}}{Area_l}+1),\ln \text{UpPay}_i,1[Large_i],1[Medium_i],1[Small_i]\}$.

Assumption 3 ***CI3*** *Conditional on bidder-license characteristics X_{il} and bidding decisions Bid_{ilt}, ξ_{il} has zero expected mean: $E[\xi_{il}|\boldsymbol{X}_{il},Bid_{ilt}]=0$*

With **Assumption 2** and **Assumption 3**, the conditional moment inequality in Equation 30 can be rewritten as

$$E[1[Bid_{ilt}=1]\overline{B}_{ilt}^{Bid=1}(\theta)-1[Bid_{ilt}=0]\overline{B}_{ilt}^{Bid=0}(\theta)|Bid_{ilt},\mathbf{X}_{il}]\geq 0 \tag{31}$$

Similar to how we estimate complementarity, we select instruments and transform conditional inequalities to their unconditional counterparts. The estimates of the $\theta's$ minimize the following criterion function.

$$\begin{aligned}Q(\theta)= \quad & \textstyle\sum_{z(\mathbf{X}_{il},Bid_{ilt})}\min\{z(\mathbf{X}_{il},Bid_{ilt})\times\\ &(1[Bid_{ilt}=1]\overline{B}_{ilt}^{Bid=1}(\theta)-1[Bid_{ilt}=0]\overline{B}_{ilt}^{Bid=0}(\theta)),0\}^2\end{aligned} \tag{32}$$

Again, we select instruments based on different quantiles of the explanatory variables. We interact Bid_{ilt} with these instruments because $Bid_{ilt} = 1$ represents a lower bound of stand-alone values while $Bid_{ilt} = 0$ represents an upper bound. We bootstrap at bidder-license level to get confidence intervals.

5.4.2 Estimates of Stand-alone Values

Table 6: Estimation of Stand-alone Values (in million \$)

	(1)	(2)	(3)	(4)	(5)
	All	All	Large	Medium	Small
θ_1	187	204	402	54	65
$\ln(\frac{Pop_c}{Area_c} + 1)$	[119,272]	[125,387]	[274,510]	[21,153]	[-63,252]
θ_2	69	32	-52	-53	6810
$\ln(\frac{Tower_{ic}}{Area_c} + 1)$	[36,94]	[-61,88]	[-66,41]	[-171,28]	[-1251,26380]
θ_3	-11	-1	-483	6	-4
$\ln(UpPay)$	[-17,-5]	[-251,11]	[-2257,695]	[-1,11]	[-16,4]
θ_4		-4	9863		
$1[Large_i]$		[-218,4995]	[-14347,46157]		
θ_5		-162		48	
$1[Medium_i]$		[-385,4026]		[-220,311]	
θ_6		-238			4337
$1[Small_i]$		[-479,3741]			[-35608,12001]
# obs (bidder-license)	2370	2370	602	664	1054

Note: all variables are interacted with Pop_c and $Bandwidth_c$.

Table 6 reports estimation results for the stand-alone values. Column (1) and (2) reports the estimates of stand-alone values with full sample. Column (3), (4) and (5) are estimation results for large, medium and small bidders respectively. When we estimate full sample/large bidder/medium bidder/small bidder, we use $\beta = 194/918/222/120$ million dollars respectively. According to the estimates in column (2), if the population density in all counties increases by 1 unit, the sum of all county level stand-alone values with a bandwidth of 1MHz will increase by \$204 million. If the tower density in all counties increase by 1 unit, the sum of all county level stand-alone values with a bandwidth of 1MHz will increase by \$32 million. If we hold constant license characteristics, a large bidder will value the stand-alone value more than smaller bidders, which is consistent with our belief. These estimates, however, are not statistically significant. Overall, heterogeneity at the license level, including bandwidth, population, population density, and cellphone tower density, plays a much larger role in bidders' stand-alone values.

5.5 Goodness of Fit

In this subsection, we compare bidders' predicted bidding decisions with their observed bidding decisions. Predicted bidding decisions are computed as follows: we first predict the stand-alone values of the licenses according to the estimates in Table 6. Then, for a bidder in a license-round, we compare the expected marginal contribution of the license (computed using Equation (4) and complementarity estimates in Table 5) with the minimum acceptable bid. If the expected marginal contribution of the license is higher than the minimum acceptable bid, we predict that the bidder will bid on the license in this round. Otherwise, the bidder will not bid.

Table 7: Goodness of Fit

	Data				
	Not Bid		Bid		
Predicted	Frequency	Percentage	Frequency	Percentage	Total
Not Bid	92,965	62.6	6,408	4.3	99,373
Bid	43,839	29.5	5,233	3.5	49,072
Total	136,804	92.2	11,641	7.8	148,445

Table 7 how well our predicted bidding fits observed data. Our empirical analysis is based on a total of 148,445 (bidder-license-round) inequalities. Our model predictions satisfy $66\% = (62.6\% + 3.5\%)$ of these inequalities.

6 Counterfactual Analysis

In the counterfactual analysis, we compare the magnitude of the exposure problem, bidder surplus and FCC revenue as well as license allocation to different types of bidders at the end of the auction under alternative package designs. As the example in our introduction illustrates, the welfare effect of package bidding depends on the distribution of bidder valuations and the size of license complementarity. In this section, we first describe our metrics to measure the incidence of the exposure problem, then explain how we conduct simulations based on our estimates, and lastly evaluate welfare trade-offs under alternative designs of package bidding.

6.1 Two Types of Exposure Problems

We characterize two types of exposure problems: In any round for any bidder, if a license in the bidder's provisional winning set does not contribute value to its ultimate winning set, we call it the "overbidding" exposure problem; in any round for any bidder, if a license that could have contributed value to the ultimate winning set is not in the provisional winning set, we call it the "underbidding" exposure problem.

When a bidder bids on a license in a round, it must have expected that this bidding decision is profitable. However, this bidder may not win all other licenses it expected to win and, in turn, the bidder may find it unprofitable to have this license at the end of the auction. In this case, the bidder may regret its bidding decision on this license, namely the "overbidding" exposure problem. The magnitude of exposure from the set $\mathbf{W}_{it}$ on $\mathbf{W}_{iT}$, is

$$EP^1(\mathbf{W}_{it}, \mathbf{W}_{iT}) \quad = \quad -\sum_{l \in \mathbf{W}_{it}} \min\{v_{il} + \beta_i \sum_{l' \in (\mathbf{W}_{iT} \setminus l)} \tau(l, l') - P_l, 0\} \tag{33}$$

where $v_{il} + \beta_i \times \sum_{l' \in (\mathbf{W}_{iT} \setminus l)} \tau(l, l')$ is the expected marginal contribution of license l towards the ultimate winning set $\mathbf{W}_{iT}$. If this expected marginal contribution is lower than the price, bidder i should not have won license l and the difference between this contribution and price is the magnitude of "overbidding" exposure problem from this license. We then aggregate over the bidders' provisional winning set in this round to calculate the total "overbidding" exposure problem in this round.

When a bidder does not bid on a license in a round, it must have expected that it is unprofitable to bid on this license in this round. Again, the set of licenses the bidder think it can win may not be the same as the ultimate winning set of the bidder. The bidder may regret that it does not bid on this license. This is the "underbidding" exposure problem. The magnitude of exposure from the set $\mathbf{W}_{it}$ on $\mathbf{W}_{iT}$, is

$$EP^2(\mathbf{W}_{it}, \mathbf{W}_{iT}) \quad = \quad \sum_{l \notin \mathbf{W}_{it}} \max\{v_{il} + \beta_i \sum_{l' \in (\mathbf{W}_{iT} \setminus l)} \tau(l, l') - P_l, 0\} \tag{34}$$

If this expected marginal contribution of license l towards the ultimate winning set $\mathbf{W}_{iT}$ is higher than the price, bidder i regrets not having won license l. The difference between this expected marginal contribution and price is the magnitude of the "underbidding" exposure problem. We then aggregate over the bidders' provisional winning set in this round to calculate the total "underbidding" exposure problem in this round.

6.2 Benchmark Model

In this simulated auction, we assume away eligibility restrictions and activity requirements to simplify bidding decisions. These FCC rules are set to induce straightforward bidding in spectrum auctions. Our simulated bidding strategies already abide by these rules. We also do not intend to let budget constraint to affect our simulation results as our model does not incorporate this constraint. Therefore, our results on different types of bidders are driven by the difference in their valuations, not by their budget constraints. To have a clean experiment, we restrict our analysis to the bidders that eventually win at least one license in Block B, the block with the smallest geographic delineation. In total, we have 79 bidders bidding on 734 licenses.

6.3 Benchmark Simulation Setup

Our simulation need to use our estimated primitives in the model: the complementarity value between any two licenses (common to all bidders of the same type) and the stand-alone value of each license for each bidder. For any pair of licenses, we can calculate the complementarity index according to Equation 11. We the use the value of complementarity effect as reported in the baseline estimation results in Table 5. Below we illustrate how we simulate the stand-alone value of a license for abide. First, we obtain a set of bidder-license-round specific lower bounds of the stand-alone values of a license for a bidder from Equation (15), for all rounds when the bidder bids on this license. We select the maximum of all these bidder-license-round specific lower bounds as the lower bound of the stand-alone value of this license for this bidder. Second, we obtain a set of bidder-license-round specific upper bounds of stand-alone values of a license for a bidder from Equation (17), for all rounds when a bidder does not bid on this license.[39] We select the minimum of all these bidder-license-round specific upper bounds as the upper bound of the stand-alone value of this license for this bidder. Combining these two steps, we obtain a lower bound and an upper bound of the stand-alone values for each bidder on each license.[40] We then simulate bidders' stand-alone values from a uniform distribution between the lower bound and upper bound of the stand-alone values of this license.[41] Based on the value of complementarity between all pairs of

---

[39] We assume the error term in this equation to be zero in simulations.

[40] When the lower bound is greater than the upper bound, we replace the upper bound with the lower bound. For a few negative stand-alone values, we replace them with zero.

[41] We may simulate stand-alone values from Table 6. However, since we use semi-parametric approach to estimate the parametrization in stand-alone values, we do not know the distribution of the residual terms. Therefore, we can not draw stand-alone values from our estimates of stand-alone values.

licenses and the stand-alone values of all license for all bidders, we proceed to simulate the bidding decisions of the bidders in the auction.

6.4 Simulation Algorithm

We describe how we simulate the entire auction in this subsection. In any round t, bidder i observes its provisional winning set $\mathbf{W}_{it-1}$. The bidder forms expectation on the probability of winning license l: $Pr(l \in \mathbf{W}_{iT}|h_{it})$ according to the belief formation functions in Equation (8) and Equation (9). Bidder i determines its bidding set in round t: $\mathbf{B}_{it}$, which maximizes its expected profit this round. Ideally, we want to compute the expected payoffs associated with all bidding strategies, that is, any possible set of licenses. For any bidder i, there are a total of 2^{734} possible bidding strategies in round t. It is computationally infeasible to evaluate the expected profit from all bidding strategies and find the optimal bidding set.

To solve this problem, we transform the profit-maximization problem into a search for the fixed points of the necessary conditions. This transformation simplifies our problem from a set with 2^{734} bidding possibilities to a set of fixed point of the necessary conditions, which has a much smaller dimension. Since all licenses are complements, if a bidder bids on any license, the marginal contribution of all other licenses towards the bidding set will increase. That is, the set of licenses display supermodularity property. We follow Jia (2008) and make use of supermodularity property to compute for the minimum bidding set and maximum bidding set of the bidders in any round.

The intuition is that, if a bidder bids on a license even though it does not win any other licenses, the optimal bidding set must include this license. On the other hand, if a bidder does not bid on a license even though it wins all other licenses, the optimal bidding set must not include this license. So we will compute for the set of licenses which the bidder should always bid on (the minimum bidding set) and the set of licenses that a bidder should potentially bid on (all licenses minus the set of licenses the bidder should not bid on, or the maximum bidding set). The optimal bidding set of the bidder should be in between the minimum bidding set and maximum bidding set.

Below we describe how we determine the minimum bidding set in round t. In any round t, the initial bidding set is an empty set and we include licenses into this bidding set sequentially. In the first iteration, we compute for the expected marginal contribution of all licenses towards its provisional winning set in round $t-1$. The bidding set in iteration 1 includes those licenses whose marginal contributions are higher than their current minimum acceptable bids. In the second iteration, we compute for the expected marginal contributions of all licenses towards the provisional winning set in round $t-1$ and the bidding set in iteration 1. The bidding set in iteration 2 includes all licenses whose marginal contribution is higher than the current minimum acceptable bids. Similarly, in the kth iteration, we compute for the expected marginal contributions of all licenses towards the provisional winning set in round $t-1$ and the bidding set in iteration $k-1$. The bidding set in iteration k includes all those licenses whose marginal contributions are higher than the current minimum acceptable bids. We iterate this sequence until the bidding set converges and the converging set will be the minimum bidding set of the bidder in round t. This sequence will converge within 734 iterations.

We determine the maximum bidding set in round t using a similar method. The initial bidding set includes all licenses and we delete licenses from this set sequentially. In any round t and iteration k, we compute for the expected marginal contributions of all licenses towards the provisional winning set in round $t-1$ and the bidding set in iteration $k-1$. A bidder's bidding set in iteration k includes all licenses whose marginal contribution are higher than the current minimum acceptable bids.

Bidders submit their bids simultaneously. If there is only one bidder bidding on a license, the bidder will become the provisional winner of the license this round. When there are more than one bidders bidding on a license, the winner will be randomly determined and every bidder has equal probability of being the winner.[42] Once a bidder becomes the provisional winner of a license, it will stop bidding on this license and only reconsiders whether to bid on this license after the round when another bidder makes a new bid on the license. The minimum acceptable price of a license increases by 10% if at least one bidder bids on this license in a round. The auction ends when no bidder places a new bid. Withdraws are not allowed in this simulation.

6.5 Counterfactual Simulations

We simulate the benchmark model and four counterfactuals with different package designs 200 times. In each simulation, we redraw a set of bidder's stand-alone values on all licenses.

We run each simulation with both minimum and maximum bidding sets: a simulation where all bidders' bidding sets are their minimum bidding sets and a simulation where all bidders' bidding sets are their maximum bidding sets. We then take average of different metrics across these simulations and report them in tables. The metrics include the magnitude of the exposure problem, bidder surplus, FCC revenue, and final license allocations

---

[42] The winner selection rule is the same as the rule in the FCC auctions.

under different package designs.

Benchmark Simulation

In the benchmark simulation, we simulate the performance of 79 bidders bidding on 734 *a la carte* CMA licenses in block B. There is no package offered in the simulated auction.

Pure BEA and REA Package

We conduct two counterfactual experiments in which we only allow predefined license packages. There is no *a la carte* CMA licenses offered in the simulated auction. In the first counterfactual experiment, we package all CMA licenses into 176 BEA packages.[43] In the second counterfactual experiment, we package all CMA licenses into into 6 REA packages.[44]

Mixed BEA and REA Package

In the last two counterfactuals, we allows for mixed package in the auction. Both *a la carte* CMA licenses and predefined (BEA and REA) packages of CMA licenses are for sale. A bidder can assemble *a la carte* CMA licenses and packages together. Suppose a package contains three licenses: A, B and C. A bidder decides whether to bid on A, B and C separately or bid for the entire package (let's call it package ABC) based on which yields higher expected payoffs. FCC decides whether to sell the three licenses as a package or sell license A, B, and C separately. FCC first evaluates its revenue selling package ABC and selling *a la carte* licenses A, B, and C. If the revenue from package ABC is higher, FCC will sell the three licenses as a package. Otherwise, FCC will sell these licenses separately.

6.6 Simulation Results

6.6.1 The Exposure Problem

Table 8 (9) reports the magnitude of bidders' exposure problem at the end of the auction if all bidders bid on their minimum (maximum) bidding sets in all rounds of the auction. In each panel, the first three rows report respectively the magnitudes for large, medium, and

---

[43] The BEA licenses defined by us in the simulated auction are not exactly the same as the BEA license defined by FCC. This is because several BEA licenses may share one CMA license based on FCC delineation. When this happens, we assign a CMA to the larger BEA.

[44] These 6 packages are the REA licenses 1 to 6 sold in block C, Auction 73, which cover the U.S. continent. REA license 7 to 12 cover Alaska, Hawaii, Guam and the Northern Mariana Islands, Puerto Rico and the U.S. Virgin Islands, American Somoa, and Gulf of Mexico.

small bidders. The last row in each panel reports the sum value of the first three rows. The simulation results in the minimum bidding set and maximum bidding set are quite similar.

Table 8: Exposure Problem in the Last Round: Minimum Bidding Set (in million dollars)

		Minimum Bidding Set				
		Benchmark	Pure Package		Mixed Package	
		CMA	BEA	REA	BEA	REA
“Overbidding”	Large Bidders	0.00	0.92	0.00	15.79	28.96
	Medium Bidders	0.00	0.00	0.00	0.00	0.00
	Small Bidders	0.00	0.00	0.00	0.00	0.00
	Sum	0.00	0.92	0.00	15.79	28.96
“Underbidding”	Large Bidders	515.12	317.15	95.85	87.13	117.62
	Medium Bidders	22.71	9.93	0.00	0.25	0.00
	Small Bidders	0.08	0.00	0.00	0.00	0.00
	Sum	537.91	327.08	95.85	87.38	117.62
Total Exposure	Large Bidders	515.12	318.07	95.85	102.92	146.58
	Medium Bidders	22.71	9.93	0.00	0.25	0.00
	Small Bidders	0.08	0.00	0.00	0.00	0.00
	Sum	537.91	328.00	95.85	103.17	146.58

Package bidding may alleviate or exacerbate the exposure problem. Package bidding produces two effects on the exposure problem: on the one hand, it eliminates the within-package exposure problem, because bidders will always win all licenses within a package or nothing at all. On the other hand, it may increase the between-package exposure problem, because the packages are larger than individual licenses and the risk of not obtaining several packages together is higher. From reported results in Table 8 and 9, we can see that the effects of package bidding on the exposure problem depend on package format and package size.

Table 8 and 9, taken together, show that most package bidding regimes alleviate the exposure problem, especially the “underbidding” exposure problem. Under mixed bidding, the reduction in “underbidding” exposure risks, however, is at the cost of sizable increases in “overbidding” exposure risks. In fact, when packages cover very large geographic areas under mixed package (REA package), the “overbidding” risks rise dramatically, leading to an exposure problem at roughly the same magnitude as the benchmark case (see last column in Table 9). Pure package, on the contrary, alleviate the exposure problem more consistently, no matter whether the package size is small (BEA) or large(REA). The effect of package size seems to be larger than the effect of package format. Large package size (REA) reduces

Table 9: Exposure Problem in the Last Round: Maximum Bidding Set (in million dollars)

		Maximum Bidding Set Benchmark	Pure Package		Mixed Package	
		CMA	BEA	REA	BEA	REA
"Overbidding"	Large Bidders	1.73	3.09	35.06	62.07	327.26
	Medium Bidders	2.30	0.00	0.00	0.00	0.00
	Small Bidders	0.02	0.00	0.00	0.00	0.00
	Sum	4.05	3.09	35.06	62.07	327.26
"Underbidding"	Large Bidders	446.25	260.76	85.06	94.23	78.46
	Medium Bidders	19.30	9.48	0.00	0.00	0.00
	Small Bidders	0.01	0.00	0.00	0.00	0.00
	Sum	465.55	270.24	85.06	94.23	78.46
Total Exposure	Large Bidders	447.97	263.85	120.11	156.30	405.73
	Medium Bidders	21.59	9.48	0.00	0.00	0.00
	Small Bidders	0.03	0.00	0.00	0.00	0.00
	Sum	469.60	273.33	120.11	156.30	405.73

the exposure problem significantly under pure package, but such reduction effect is not clear under mixed package. Small package size (BEA) produces lower variances in such effect.

More importantly, different types of bidders bear different burden when package policies change. Decomposing a bidder's ex-post regret, we can see that the largest chunk of the exposure problem under benchmark simulation comes from the ex-post regret from large bidders' premature withdrawal. Package bidding, especially mixed package, changes this scenario drastically. Both tables show large bidders have very high "overbidding" exposure problem and very low "underbidding" exposure problem under mixed package. This means that large bidders bid more aggressively: they win some licenses that they do not want to win at the end of the auction (thus high "overbidding" regret) and lose few licenses they intend to win at the end of the auction (thus low "underbidding" regret). The change in large bidders' bidding behavior is crucial in determining auction outcomes, as will be discussed in the next two subsections.

6.6.2 Bidder Surplus and FCC Revenue

Table 10 (11) reports the bidders' surplus and FCC revenue at the end of the auction if all bidders bid on their minimum (maximum) bidding sets in all rounds of the auction. In each panel, the first three rows report the values for large, medium, and small bidders, respectively. The last row in each panel reports the sum value of the first three rows. The very bottom row of the table sums up bidder surplus and FCC revenue to report the metric for social surplus under different simulations. Again, the simulation results in the minimum bidding set and maximum bidding set are quite similar.

Table 10: Bidders' Surplus and FCC Revenue: Minimum Bidding Set (in billion dollars)

		Minimum Bidding Set				
		Benchmark	Pure Package		Mixed Package	
		CMA	BEA	REA	BEA	REA
Bidder Surplus	Large Bidders	9.45	6.93	3.09	5.09	2.61
	Medium Bidders	0.28	0.01	0.00	0.00	0.00
	Small Bidders	0.42	0.30	0.00	0.15	0.00
	Sum 1	10.15	7.24	3.09	5.24	2.61
FCC Revenue	Large Bidders	3.09	7.00	12.35	9.89	12.76
	Medium Bidders	0.52	0.06	0.00	0.00	0.00
	Small Bidders	0.14	0.28	0.00	0.44	0.00
	Sum 2	3.74	7.34	12.35	10.33	12.76
Social Surplus	Sum 1 + Sum 2	13.89	14.58	15.44	15.57	15.37

Table 11: Bidders' Surplus and FCC Revenue: Maximum Bidding Set (in billion dollars)

		Maximum Bidding Set				
		Benchmark	Pure Package		Mixed Package	
		CMA	BEA	REA	BEA	REA
Bidder Surplus	Large Bidders	9.26	7.02	2.61	4.23	1.29
	Medium Bidders	0.20	0.00	0.00	0.00	0.00
	Small Bidders	0.42	0.30	0.00	0.15	0.00
	Sum 1	9.88	7.32	2.61	4.38	1.29
FCC Revenue	Large Bidders	3.77	7.32	12.69	11.05	13.94
	Medium Bidders	0.48	0.04	0.00	0.00	0.00
	Small Bidders	0.14	0.29	0.00	0.43	0.00
	Sum 2	4.39	7.65	12.69	11.49	13.94
Social Surplus	Sum 1 + Sum 2	14.27	14.97	15.30	15.86	15.22

The FCC revenue in the benchmark simulations ($3.74 to 4.39 billion dollars) are close to the actual FCC revenue in block B ($9.14 billion). Table 10 and Table 11 show that when FCC exercises package bidding, FCC has much to gain but bidders have much to lose. All four types of package bidding increase total social surplus slightly, but the distribution between FCC and bidders are quite uneven. First, FCC receives a huge gain in revenue under mixed package than under benchmark. This is because large bidders bid much more aggressively under mixed package, especially under REA mixed package. This aggressive bidding behavior push final prices up, benefiting FCC revenue while hurting bidders who bear the burden of high prices.

Second, FCC also does much better under mixed package than under pure package. This because mixed package endows FCC a much more flexible tool, allowing FCC to take *a la carte* license payoff or packaged license payoff, whichever yields FCC a higher revenue.

Third, FCC has much more to gain (and bidders have much to lose) when packages cover large geographic areas. This is consistent with the example in the introduction. When there is substantial heterogeneity in stand-alone values, it may not be a valid practice to offer large packages to bidders. This is because a package may contain a license which bidder 1 values a lot but bidder 2 does not value and another license which bidder 2 values a lot but bidder 1 does not value. A package that includes both licenses wipes out the valuation heterogeneity and does not award the individual license to the bidder who has the highest value for it. Bidders then compete in bidding for low-value packages, generating low bidder surplus.

Combining results on both bidder surplus and FCC revenue, we find that switching from *a la carte* bidding to mixed package only slightly increases social welfare while FCC benefits substantively from the switch.

6.6.3 Final License Allocation to Different Types of Bidders

Table 12 (13) reports final license allocation at the end of the auction if all bidders bid on minimum (maximum) bidding set in all rounds. The five columns are the same as the last table. The first row reports the percentage of packages sold as bundles. The second to the fourth rows report the population-weighted market shares of large, medium, and small bidders respectively. The fifth row reports the Hirschman Herfindahl Index (HHI) based on population-weighted market shares. The sixth to eighth rows report the unweighted market shares of large, medium, and small bidders respectively. The last row reports the unweighted market shares.[45]

Bidders do bid on packages when they have a choice. Under BEA mixed package policy, roughly 75% of licenses are sold as packages. In contrast, under REA mixed package policy, 100% of the licenses are sold as packages. Large bidders win more licenses under both mixed and pure package regimes. On the contrary, medium bidders lose almost all market shares,

---

[45] HHI based on population-weighted market shares is calculated as the sum of squared population shares won by each firm; HHI based on unweighted market shares is calculated as the sum of squared license shares won by each firm.

Table 12: Licence Allocation: Minimum Bidding Set

	Minimum Bidding Set				
	Benchmark	Pure Package		Mixed Package	
	CMA	BEA	REA	BEA	REA
Percentage of Packages	0	100	100	74.52	100
Market Share and HHI (Population Weighted)					
Market Share (Large)	89.14	95.73	100.00	96.46	100.00
Market Share (Medium)	8.02	0.65	0.00	0.00	0.00
Market Share (Small)	2.84	3.62	0.00	3.54	0.00
HHI (population)	6777	7495	9582	8502	9362
Market Share and HHI (Unweighted)					
Market Share (Large)	98.75	98.99	100.00	99.63	100.00
Market Share (Medium)	1.11	0.43	0.00	0.00	0.00
Market Share (Small)	0.14	0.58	0.00	0.37	0.00
HHI (licenses)	8637	8543	10000	9352	9778

Table 13: Licence Allocation: Maximum Bidding Set

	Maximum Bidding Set				
	Benchmark	Pure Package		Mixed Package	
	CMA	BEA	REA	BEA	REA
Percentage of Packages	-	-	-	75.06	100.00
Market Share and HHI (Population Weighted)					
Market Share (Large)	89.74	95.92	100.00	96.46	100.00
Market Share (Medium)	7.42	0.46	0.00	0.00	0.00
Market Share (Small)	2.84	3.62	0.00	3.54	0.00
HHI (population)	7685	8657	9152	9647	9023
Market Share and HHI (Unweighted)					
Market Share (Large)	98.20	99.10	100.00	99.63	100.00
Market Share (Medium)	1.65	0.32	0.00	0.00	0.00
Market Share (Small)	0.15	0.58	0.00	0.37	0.00
HHI (licenses)	9310	9490	9556	9913	9417

and small bidders lose almost all market shares when FCC adopts REA mixed package. This is because medium and small bidders value complementarity much less than large bidders do and as a result they are not able to compete with large bidders in prices under package bidding. Switching from *a la carte* bidding to mixed package, a small number of bidders win almost all licenses and HHI increases substantially. The implications of these results, as they concern future competitive landscape in the cellphone market, are not included in our short-run welfare assessment at the conclusion of the auction. A key policy goal of FCC is to create and maintain a competitive market place for the telecommunications industries. The goal is embodied by the steep bidding credit FCC offers to small bidders in spectrum auctions.[46] For this goal, policies favoring small to medium bidders are better than those favoring large, incumbent bidders.

7 Conclusion

In this paper, we evaluate the magnitudes of license complementarity and, in turn, the welfare tradeoffs of alternative package bidding policies in FCC spectrum auctions. The key to our approach is to construct moment inequalities derived from an incomplete model characterizing bidder behaviors. We find that heterogeneity of bidders play a large role is their valuation of license complementarity: A large bidder, such as AT&T, values 1 MHz nation wide license at more than 4 times than a medium bidder does, and more than 6 times than a small bidder does. Under mixed package bidding policy, these large bidders gain more freedom because they can assemble *a la carte* licenses and packages to bid on with less concern about only winning isolated patches of licenses. As a result, they bid more aggressively and wins more licenses, giving FCC a large revenue boost while effectively driving medium and small bidders out of license allocation. We recommend policy makers exercise caution when considering package bidding as a policy remedy. Although having the benefit of reducing bidders' *ex post* regret, package bidding may favor license allocation to more powerful players in the telecommunication market, which hampers the development of a more leveled field for all types of competitors.

---

[46] Another notable example of this goal is the 1996 Telecommunications Act, which aimed to promote competitive entry into the local telephone market. The act eliminates a state's authority to erect legal entry barriers to local telephone markets. The act also mandates incumbents to offer free interconnections and to lease their network to any new entrant(Fan and Xiao (2015)).

Appendix

1) Bidders Bid on and Stop Bidding on Multiple Licenses at the Same Time

Table A1 reports the number of licenses that a bidder bids or stops bidding on for a given round of a block. We restrict to a sub-sample that starts from round 20 and that only contains bidders who has placed at least one bid in a round in the license's associated block. There are 5,358 such bidder-block-round observations. As shown in the left panel, bidders often place multiple bids in a block. As shown in the right panel, it is a frequent event that a bidder stops bidding on multiple license at the same time. A bidder bids on a maximum of 197 licenses in a block-round and stops bidding on maximum of 21 licenses in a block-round. This is clear evidence of bidders' expected complementarity over multiple licenses.

Table A1: Multiple Bids Placing and Bidding Stops by a Bidder in a Round

Place Bids			Stop Bidding		
# of License	Freq	Percentage	# of License	Freq	Percentage
			0	5001	92.87
1	3219	59.78	1	296	5.5
2	922	17.12	2	50	0.93
3 to 5	745	13.83	3 to 5	18	0.33
6 to 10	275	5.11	6 to 10	14	0.26
11 to 20	119	2.21	11 to 20	4	0.07
21 to 197	105	1.95	21	2	0.04
Total	5385	100	Total	5385	100

2) BA1+BA2 $\Rightarrow$ BA3

Assumption 4 ***BA3*** *If* $l \in \boldsymbol{B}_{it}$, $l' \notin \boldsymbol{B}_{it}$ *and* $l' \notin \boldsymbol{W}_{t-1}$, $\Delta V_i(l, \boldsymbol{W}_{it-1} \cup \boldsymbol{B}_{it}, Pr(\boldsymbol{W}_{iT})) - P_{lt} \times Pr(l \in \boldsymbol{W}_{iT}) > \Delta V_i(l', \boldsymbol{W}_{it-1} \cup \boldsymbol{B}_{it} \backslash l, Pr(\boldsymbol{W}_{iT})) - P_{l't} \times Pr(l' \in \boldsymbol{W}_{iT})$ *if a bid on* l' *is also under eligibility constraint.*

BA3 states that, if a bidder bids on license l but not license l', the marginal contribution of license l towards the bidder's winning set is higher than marginal contribution of license l' towards the bidder's winning set.[47] Below, we show that if **BA1** and **BA2** hold, **BA3** is

---

[47] Yeo (2009) uses **BA3** for estimation although her model ignores bidder belief during the Auction. The **BA3** here is the counterpart of the **BA3** in her paper.

automatically satisfied.

BA1 is equivalent to

$$v_{il} \times Pr(l \in \mathbf{W}_{iT}) + \beta_i \times \sum_{l'' \in (\mathbf{W}_{it-1} \cup \mathbf{B}_{it} \backslash l)} \tau(l, l'') \times \quad Pr(l \in \mathbf{W}_{iT}) \times Pr(l'' \in \mathbf{W}_{iT}) \\ \geq P_{lt} \times Pr(l \in \mathbf{W}_{iT}) \tag{35}$$

BA2 (applying to license l') is equivalent to:

$$v_{il'} \times Pr(l' \in \mathbf{W}_{iT}) + \beta_i \times \sum_{l'' \in (\mathbf{W}_{it-1} \cup \mathbf{B}_{it})} \tau(l', l'') \times \quad Pr(l' \in \mathbf{W}_{iT}) \times Pr(l'' \in \mathbf{W}_{iT}) \\ \leq P_{l't} \times Pr(l' \in \mathbf{W}_{iT}) \tag{36}$$

BA3 is equivalent to

$$v_{il} \times Pr(l \in \mathbf{W}_{iT}) + \\ \beta_i \times \sum_{l'' \in (\mathbf{W}_{it-1} \cup \mathbf{B}_{it} \backslash l)} \tau(l, l'') \times Pr(l \in \mathbf{W}_{iT}) \times Pr(l'' \in \mathbf{W}_{iT}) - P_{lt} \times Pr(l \in \mathbf{W}_{iT}) \geq \\ v_{il'} \times Pr(l' \in \mathbf{W}_{iT}) + \\ \beta_i \times \sum_{l'' \in (\mathbf{W}_{it-1} \cup \mathbf{B}_{it} \backslash l')} \tau(l', l'') \times Pr(l' \in \mathbf{W}_{iT}) \times Pr(l'' \in \mathbf{W}_{iT}) - P_{l't} \times Pr(l' \in \mathbf{W}_{iT}) \tag{37}$$

From **BA1** we have

$$v_{il} \times Pr(l \in \mathbf{W}_{iT}) + \\ \beta_i \times \sum_{l'' \in (\mathbf{W}_{it-1} \cup \mathbf{B}_{it} \backslash l)} \tau(l, l'') \times Pr(l \in \mathbf{W}_{iT}) \times Pr(l'' \in \mathbf{W}_{iT}) - P_{lt} \times Pr(l \in \mathbf{W}_{iT}) \geq 0 \tag{38}$$

From **BA2** we have

$$\begin{aligned} & v_{il'} \times Pr(l' \in \mathbf{W}_{iT}) + \\ \beta_i \times \sum_{l'' \in (\mathbf{W}_{it-1} \cup \mathbf{B}_{it} \setminus l')} & \tau(l', l'') \times Pr(l' \in \mathbf{W}_{iT}) \times Pr(l'' \in \mathbf{W}_{iT}) - P_{l't} \times Pr(l' \in \mathbf{W}_{iT}) \leq \\ & v_{il'} \times Pr(l' \in \mathbf{W}_{iT}) + \\ \beta_i \times \sum_{l'' \in (\mathbf{W}_{it-1} \cup \mathbf{B}_{it})} & \tau(l', l'') \times Pr(l' \in \mathbf{W}_{iT}) \times Pr(l'' \in \mathbf{W}_{iT}) - P_{l't} \times Pr(l' \in \mathbf{W}_{iT}) \leq 0 \end{aligned} \tag{39}$$

$$Q.E.D.$$

Combining Equation 38 and Equation 39, we can see the left hand side of Equation 37 is greater than the right hand side of Equation 37. Therefore, if **BA1** and **BA2** hold, **BA3** is automatically satisfied. Intuitively, **BA1** implies that if a bidder bids on license l, the marginal contribution of license l is positive. **BA2** implies that if a bidder does not bid on license l', the marginal contribution of license l' is negative. So the marginal contribution of license license l is greater than marginal contribution of license license l', which is **BA3**.

3) Constructing Cellphone Tower Variable

FCC maintains a cellphone tower registration database that records all tower ownership or usage information.[48] We use two data sets from this database: EN data set and RA data set. EN data set records the owner of a tower. RA data set matches each tower to a county. We count the number of towers a firm has in a county after we merge these two data sets. Lastly, we manually match firm names in the tower registration database with the identity of the bidders in the auction data to obtain the number of towers a bidder has in a county.

A drawback of the data is that the ownership changes over time but FCC does not keep records of historical database. So what we use in this paper is the tower ownership structure in late 2016 when we started the project. As Auction 73 happened in 2008, there may be changes in the number of towers a bidder has in a county in the 8 years' gap. These changes include: change of ownership, new towers, merger between bidders, etc. Still, these data are the best data we can get to measure bidders' stock of cellphone towers at different counties. We think this variable is a combination of towers that had been built by Auction 73 and towers that will be build after Auction 73, which is a good measure of bidder heterogeneity.

---

[48] The website is `http://wireless.fcc.gov/uls/`.

4) Comparison of Different Complementarity Indices

In this paper, we do not incorporate the measure of travel complementarity as in Fox and Bajari (2013) and Yeo (2009) for the following reasons: first, their measures of license complementarity are at BTA or CMA level but our measure is at county level. If we construct measures of travel complementarity at county level, there may be substantial bias in the measure of complementarity because there will be high complementarity between counties with an airport but no complementarity in the county with no airport even though the county is close to the airport. Second, we believe travel complementarity is not as important as distance complementarity. This conjecture is consistent with the empirical findings in Fox and Bajari (2013)'s empirical results, where the two travel complementarity parameters are not significant. Third, Fox and Bajari (2013) have shown that the three complementarity measures (one distance complementarity index and two travel complementarity indices) are highly correlated.

Table A2: Summary of Complementarity by Market Type

		County-level		CMA-level		CMA-level 2	
Market Type		mean	s.d	mean	s.d	mean	s.d
CMA	Within	0.415	2.526	0.415	2.526	-	-
	Between	0.003	0.047	0.003	0.047	0.004	0.130
BEA	Within	3.018	7.904	3.296	8.390	4.913	9.140
	Between	0.032	0.232	0.038	0.262	0.033	0.492
REA	Within	136.054	17.143	135.445	40.674	156.386	40.416
	Between	12.245	10.220	11.233	8.166	2.792	3.943
National license	Benchmark	1000		1000		1000	

Note: Complementarity listed in the table are complementarity indices ×1000.

Table A2 compares three measures of market complementarity. All of them are based on geographic complementarity function in Fox and Bajari (2013) when $\delta = 2$. Measure 1 is a measure at county level complementarity. This is the one we have discussed in main text and used as our empirical input. Measure 2 is a measure at CMA level complementarity. If one CMA belongs to more than one BEAs, we assume it is part of the BEA with more population. Measure 3 is also a measure at CMA level complementarity. The difference between measure 2 and 3 is that measure 3 follow Yeo (2009) and treat the distance between CMA l and l' as the minimum distance between two counties within CMA l and CMA l'. These two counties must belong to the same state. These summary statistics show that all

Table A3: Estimates of Complementarity Effect (in million $)

	(1) Full Sample	(2) Large	(3) Medium	(4) Small
β	1037	1226	500	1216
95% CI	[727,1344]	[548,1747]	[421,739]	[669,1310]
# LBs	1897	127	827	943
# Ubs	4169	33	1614	2522

Note: the unit of observation is bidder-license-(round-pairs). For each bidder-license, we match each round where the bidder place a bid with a round where the bidder does not place a bid.

three measures are very close to each other.

5) Robustness to Table 5

We have imposed two behavior assumptions on bidders: **BA1** and **BA2**. **BA1** is straightforward to establish: If a bidder bids on a license, then: 1) the marginal contribution of the license to the set of licenses that the bidder believes it will win is higher than the minimum acceptable bid of the license, and 2) the license is under the bidder's eligibility constraint. **BA2**, however, warrants extra discussion. The action of "not bid" or "stopping bidding" have more possibilities than what we allow under the "straight-forward" bidder assumption. There are three potential reasons why a bidder chooses not to bid on a license: 1) the marginal contribution of the license is lower than the minimum acceptable bid; 2) the bidding units of the license are higher than the remaining eligibility of the bidder; 3) the bidding units of the license, although lower than the remain eligibility of the bidder, are too low to satisfy FCC's activity requirement so the bidder may want to bid on other licenses instead to maintain its eligibility. Our baseline estimation (results shown in Table 5) incorporate both reason 1) and reason 2), but does not consider reason 3. In this robustness check, we add one more restriction: the bidding units of the license need to be higher than the additional units required to maintain the eligibility level of the bidder (but still lower than the unused eligibility of the bidder). Note this restriction reduces the number of observations used in estimation substantially, which is probably why we have much wider confidence intervals in Table A3.

Table A4: Estimates of Complementarity: Incorrect “Must Win” Belief (in million $)

	Case **I** (Baseline Results)			
	(1) Full Sample	(2) Large	(3) Medium	(4) Small
β	133	264	151	39
95% CI	[127,140]	[189,345]	[145,158]	[38,41]
# LBs	107651	4796	64046	38809
# Ubs	266437	10511	117285	138641
	Case **II** (Robustness Check)			
	(5) Full Sample	(6) Large	(7) Medium	(8) Small
β	34	-557	546	38
95% CI	[-54,95]	[-1509,246]	[300,908]	[4,55]
# LBs	1897	127	827	943
# Ubs	4169	33	1614	2522

Note: The unit of observation is bidder-license-(round-pairs). For each bidder-license, we match each round where the bidder place a bid with a round where the bidder does not place a bid.

6) Estimates of Complementarites under the (Incorrect) Belief of Winning Probabilities equal to 1

Table A4 is analog of Table 5 but we set a bidder’s expected probability of winning a license equal to one if the bidder places a bid on or is the provisional winner of a license.[49] As we expected, the estimates in Table A4 is substantially lower than those in Table 5. According to our model, if a bidder bids on a license while the price increases, the increase in *expected* complementarity is more than the increase in price. The fact that bidders do not win licenses with probability one and should not expect so means that the increase in *expected* complementarity must be lower than the absolute, “true” complementarity. In Table 5 we estimate the “true” complementarity, and in Table A4 what we estimate is actually the *expected* complementarity. Comparing these two tables, we see that we could underestimate complementarity if we ignore bidder belief of winning probability in a misspecified model.

With two licenses, we have four possible scenarios: win both licenses, win only license l, win only license l', win none of them. The sum of probabilities of all four scenarios equals 1. It is difficult to estimate the join probability of all scenarios when there are more licenses

---

[49] Yeo (2009) assumes this bidder belief.

due to high dimensionality. So we propose method to reduce dimensionality by estimating winning probability of each individual license.

References

Andrews, D. W., and X. Shi (2013): "Inference based on conditional moment inequalities," *Econometrica*, 81(2), 609–666.

Asker, J. (2010): "A study of the internal organization of a bidding cartel," *American Economic Review*, 100(3), 724–62.

Ausubel, L. M., P. Cramton, R. P. McAfee, and J. McMillan (1997): "Synergies in wireless telephony: Evidence from the broadband PCS auctions," *Journal of Economics and Management Strategy*, 6, 497–527.

Bomberger, C. (2007): "Spectrum auction process and integrated spectrum auction system: auction 73 and 76 700 Mhz band," *FCC document.*

Bulow, J., J. Levin, and P. Milgrom (2009): "Winning play in spectrum auctions," *National Bureau of Economic Research Working Paper No. 14765.*

Cantillon, E., and M. Pesendorfer (2006): "Combination bidding in multi-unit auctions," *CEPR Discussion Paper No. 6083.*

Chernozhukov, V., H. Hong, and E. Tamer (2007): "Estimation and confidence regions for parameter sets in econometric models," *Econometrica*, 75(5), 1243–1284.

Conley, T. G., and F. Decarolis (2016): "Detecting bidders groups in collusive auctions," *American Economic Journal: Microeconomics*, 8(2), 1–38.

Cramton, P., and J. A. Schwartz (2002): "Collusive bidding in the FCC spectrum auctions," *Contributions in Economic Analysis & Policy*, 1(1).

Doraszelski, U., K. Seim, M. Sinkinson, and P. Wang (2017): "Ownership concentration and strategic supply reduction," *National Bureau of Economic Research Working Paper No. 14765.*

Ericson, R., and A. Pakes (1995): "Markov-perfect industry dynamics: A framework for empirical work," *The Review of Economic Studies*, 62(1), 53–82.

Fan, Y., and M. Xiao (2015): "Competition and Subsidies in the Deregulated U.S. Local Telephone Industry," *the RAND Journal of Economics*, 46(4), 751–776.

FCC (2008): "Public notice: auction of 700 MHz band licenses, AU docket no. 07-157," *FCC document.*

Fox, J. T. (2010): "Identification in matching games," *Quantitative Economics*, 1(2), 203–254.

Fox, J. T., and P. Bajari (2013): "Measuring the efficiency of an FCC spectrum auction," *American Economic Journal: Microeconomics*, 5(1), 100–146.

Fox, J. T., and N. Lazzati (2017): "A note on identification of discrete choice models for bundles and binary games," *Quantitative Economics*, 8(3), 1021–1036.

Gentry, M., T. Komarova, and P. Schiraldi (2018): "Preferences and performance in simultaneous first-price auctions: a structural analysis," *Working Paper, London School of Economics.*

Gentzkow, M. (2007): "Valuing new goods in a model with complementarity: Online newspapers," *American Economic Review*, 97(3), 713–744.

Haile, P. A., and E. Tamer (2003): "Inference with an incomplete model of English auctions," *Journal of Political Economy*, 111(1), 1–51.

Hong, H., and M. Shum (2003): "Econometric models of asymmetric ascending auctions," *Journal of Econometrics*, 112(2), 327–358.

Hortaçsu, A. (2011): "Recent progress in the empirical analysis of multi-unit auctions," *International Journal of Industrial Organization*, 29(3), 345–349.

Hortaçsu, A., J. Kastl, and A. Zhang (2018): "Bid shading and bidder surplus in the us treasury auction system," *American Economic Review*, 108(1), 147–69.

Hortaçsu, A., and S. L. Puller (2008): "Understanding strategic bidding in multi-unit auctions: a case study of the Texas electricity spot market," *The RAND Journal of Economics*, 39(1), 86–114.

Jia, P. (2008): "What happens when wal-mart comes to town: an empirical analysis of the discount retailing industry," *Econometrica*, 76(6), 1263–1316.

Kawai, K., and J. Nakabayashi (2015): "Detecting large-scale collusion in procurement auctions," *SSRN Working Paper.*

Kim, S. W., M. Olivares, and G. Y. Weintraub (2014): “Measuring the performance of large-scale combinatorial auctions: a structural estimation approach,” *Management Science*, 60(5), 1180–1201.

Moreton, P. S., and P. T. Spiller (1998): “Whats in the air: Interlicense synergies and their impact on FCC broadband PCS spectrum auctions,” *Journal of Law and Economics*, 41, 677–716.

Pakes, A., J. Porter, K. Ho, and J. Ishii (2015): “Moment inequalities and their application,” *Econometrica*, 83(1), 315–334.

Wolfram, C. D. (1997): “Strategic bidding in a multi-unit auction: an empirical analysis of bids to supply electricity,” *National Bureau of Economic Research Working Paper No. 6269.*

Yeo, J. (2009): “Estimation of bidder valuations in a FCC spectrum auction,” in *Financial Intermediation Research Society Conference.*