A NOTE ON GILBREATH'S CONJECTURE

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We form a triangle with integer entries in the following way. In the first row write the prime numbers. In each following row, each entry is the absolute value of the difference of the two entries above it.

$$\begin{array}{ccccccccccccc}
2 & & 3 & & 5 & & 7 & & 11 & & 13 & & \cdots \\
& 1 & & 2 & & 2 & & 4 & & 2 & & \cdots & \\
& & 1 & & 0 & & 2 & & 2 & & \cdots & & \\
& & & 1 & & 2 & & 0 & & \cdots & & & \\
& & & & 1 & & 2 & & \cdots & & & & \\
& & & & & 1 & & \cdots & & & & &
\end{array}$$

Gilbreath's conjecture is that the entries on the left diagonal are all 1, except for the first row.

Let $A_{j,k} = |A_{j-1,k+1} - A_{j-1,k}|$. Thus if we specify $A_{0,k}$ for all $k \geq 0$ then this determines $A_{j,k}$ for all $j \geq 1, k \geq 0$.

$$\begin{array}{ccccccccccccc}
A_{0,0} & & A_{0,1} & & A_{0,2} & & A_{0,3} & & A_{0,4} & & A_{0,5} & & \cdots \\
& A_{1,0} & & A_{1,1} & & A_{1,2} & & A_{1,3} & & A_{1,4} & & \cdots & \\
& & A_{2,0} & & A_{2,1} & & A_{2,2} & & A_{2,3} & & \cdots & & \\
& & & A_{3,0} & & A_{3,1} & & A_{3,2} & & \cdots & & & \\
& & & & A_{4,0} & & A_{4,1} & & \cdots & & & & \\
& & & & & A_{5,0} & & \cdots & & & & &
\end{array}$$

If we specify $A_{0,k}$ to be the $k+1$st prime number, Gilbreath's conjecture is that $A_{j,0} = 1$ for all $j \geq 1$.

We have

$$A_{j,k} - A_{j-1,k} = |A_{j-1,k+1} - A_{j-1,k}| - A_{j-1,k}.$$

This looks like the PDE

$$u_t(t,x) = |u_x(t,x)| - u(t,x).$$

Let $u(0,x) = f(x)$. If $f'(x) \geq 0$ for all x then a solution to this PDE is $u(x,t) = e^{-t} f(x+t)$. If f is subexponential then for any x, $u(x,t) \to 0$ as $t \to \infty$. Maybe this suggests that entries in the triangular array get small at an exponential rate.

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